A posteriori Error Estimates for Numerical Solutions to Hyperbolic Conservation Laws

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Consider the Cauchy problem for a strictly hyperbolic system of conservation laws

$$u_t + f(u)_x \ = \ 0, \qquad u(0,x) \ = \ \bar{u}(x).$$

It is well known that there exists a unique entropy-weak solution, depending continuously on the initial data. Assuming small total variation, a priori estimates on the $\mathbf{L}^1$ distance between an approximate solution and the exact solution have been obtained in connection with (i) front tracking approximations, (ii) the Glimm scheme, and (iii) vanishing viscosity approximations. However, no a priori estimate is yet known for approximate solutions obtained by fully discrete schemes, such as the Lax-Friedrichs or the Godunov scheme. Therefore we focus on *a posteriori* error estimates. The result we show is the following.

Let u^{approx} be an approximate solution produced by a conservative scheme which dissipates entropy, and assume that

(i) the total variation of $u^{approx}(t, \cdot)$ is uniformly bounded,

(ii) outside a finite number of narrow strips in the domain $[0, T] \times \mathbb{R}$, the local oscillation of u^{approx} remains small.

Then the $\mathbf{L}^1$ distance

$$\|u^{approx}(T, \cdot) - u^{exact}(T, \cdot)\|_{\mathbf{L}^1(\mathbb{R})} \,.$$

is small. Our estimates do not require any regularity of the exact solution. We provide an error bound which can be applied to a wide class of approximation schemes.

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