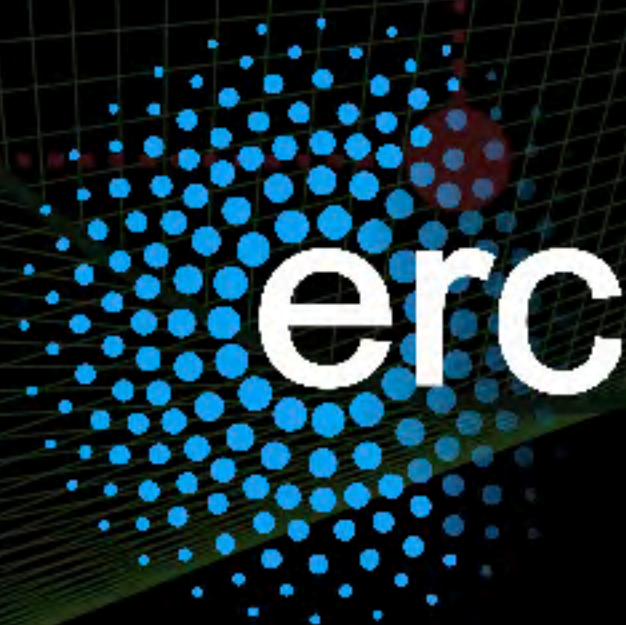


# statistical mechanics of graph neural networks

math+x

ivan dokmanić | SADA | UniBas + UIUC

november 9, 2022



University  
of Basel



(we shrunk but we will regrow)

# teaching

autumn

spring

occasionally

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pattern recognition

a practical introduction  
to data science

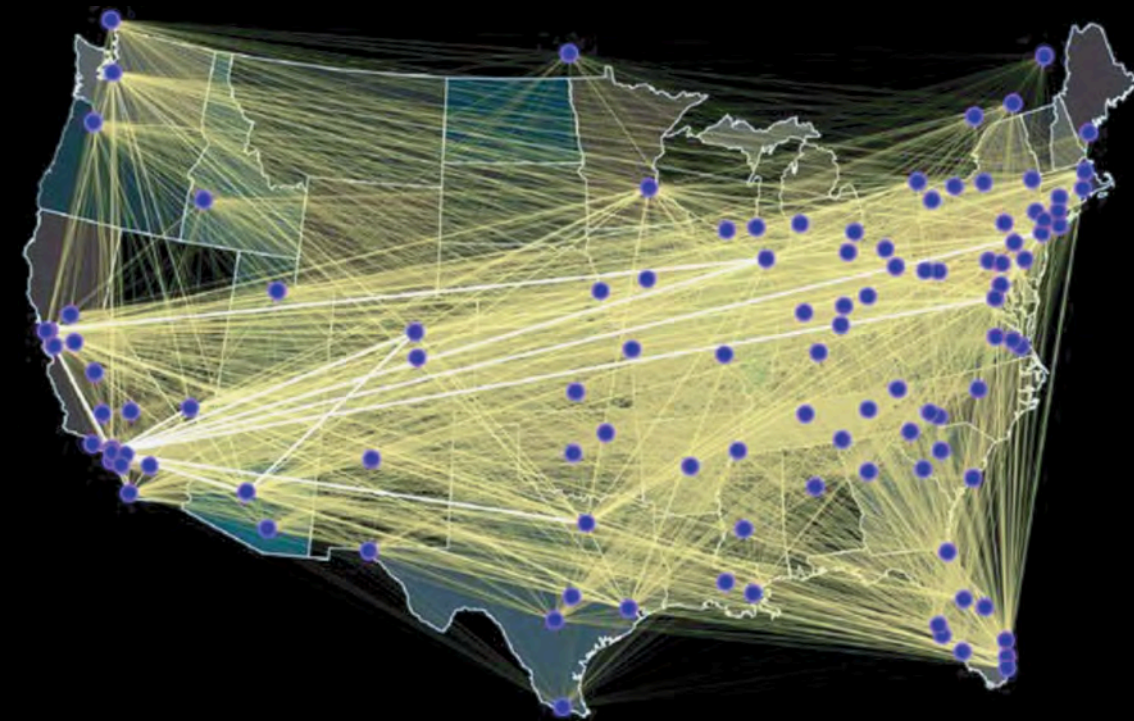
computational sustainability

mathematics of data  
science

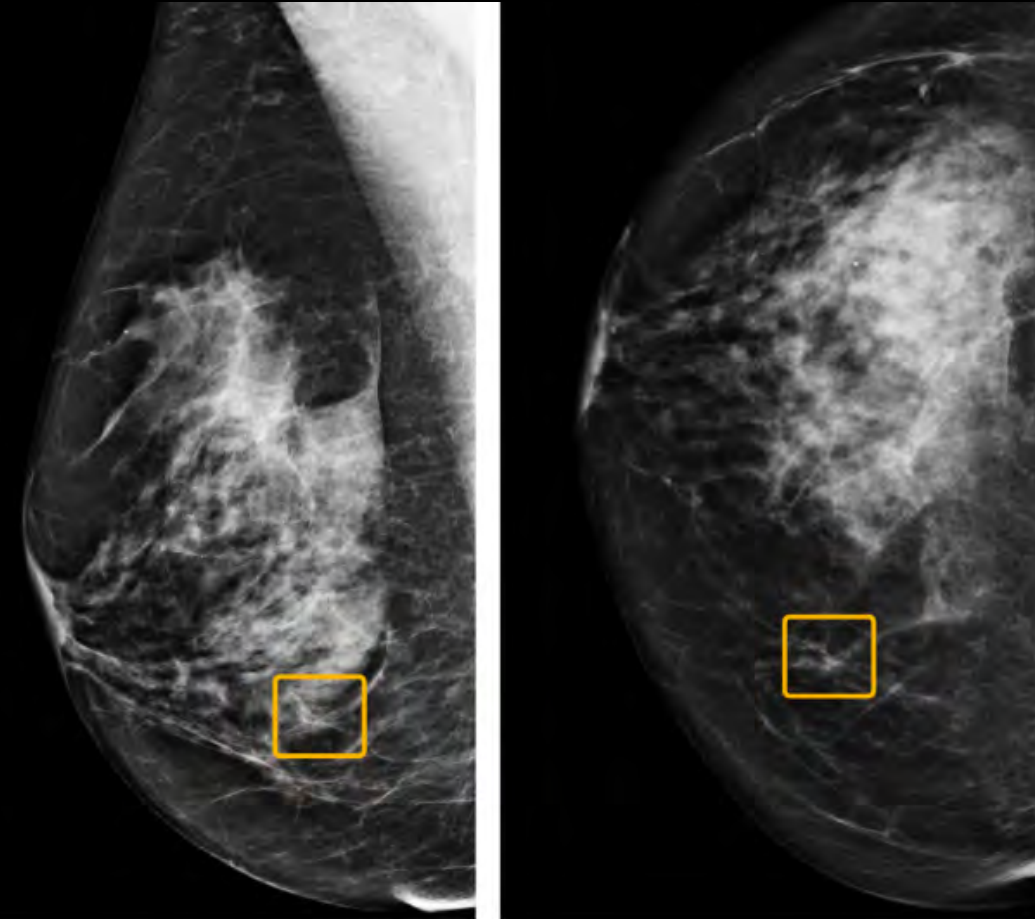
(ML on graphs)

AI 4 social good

# research: tools and theory



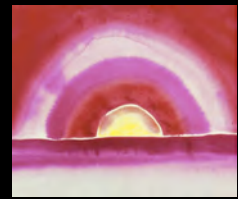
ML + physics of  
complex networks  
ML + dynamical systems



ML for imaging

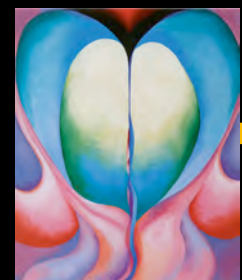
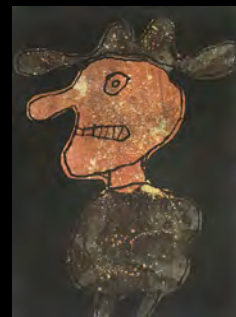
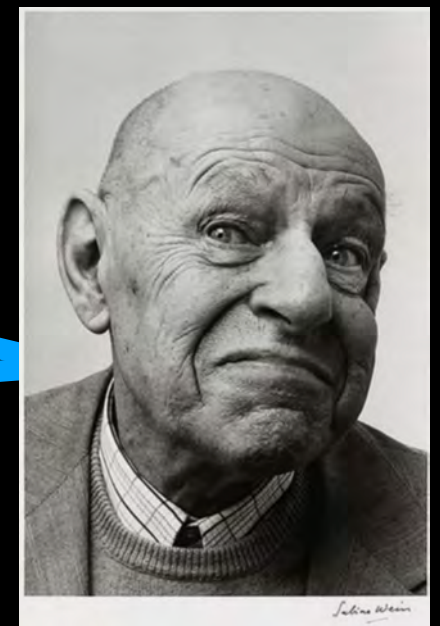


ML for Earth and  
planetary science

$\mathcal{X}$ hypothesis  $f \in \mathcal{F}$  $\mathcal{Y}$ unknown  $p_{X,Y}$ minimize  $\mathbb{P} [f(X) \neq Y]$   
 $f \in \mathcal{F}$ 

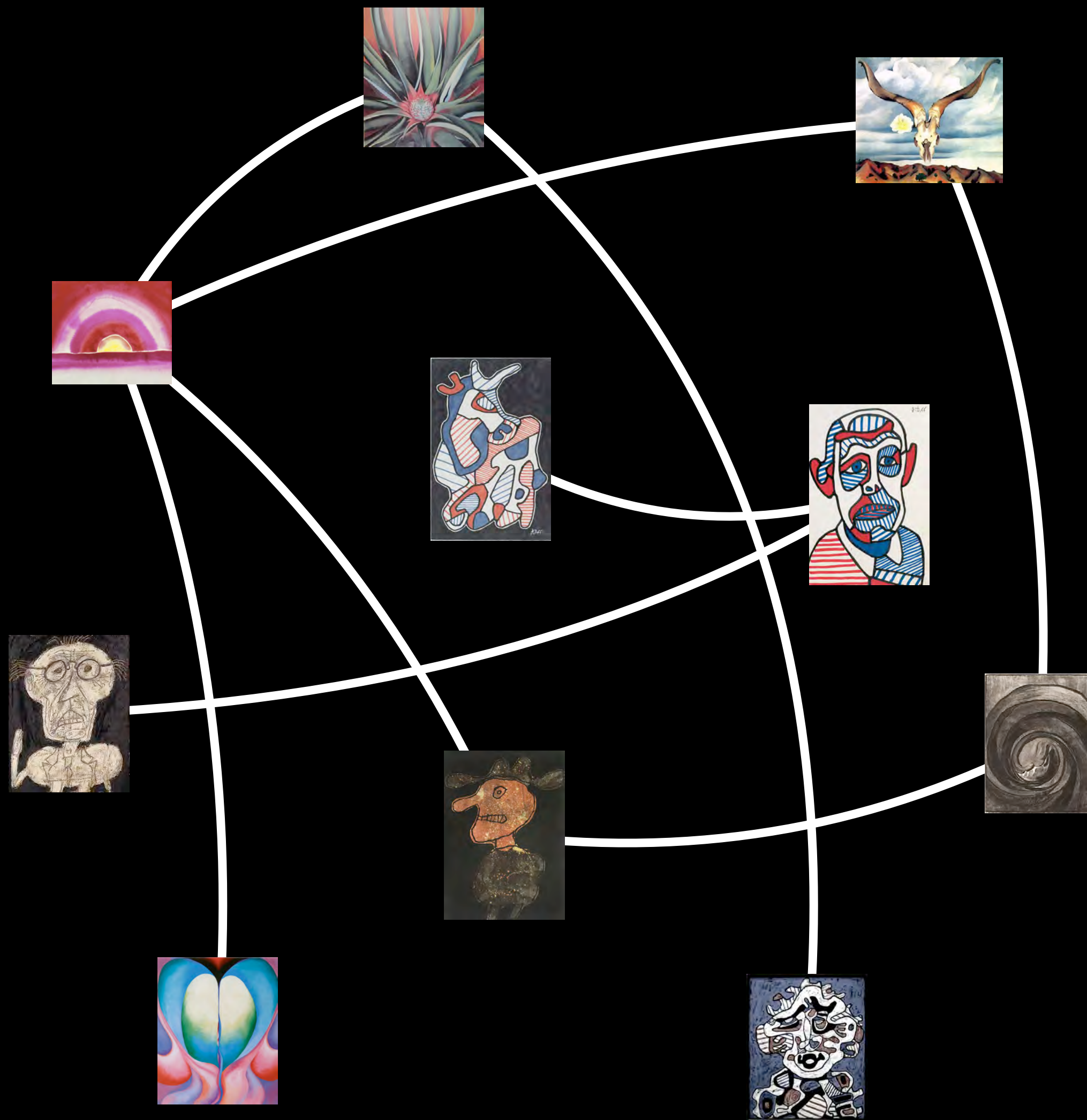
"training set"

$$\mathcal{T} = \{(x_n, y_n)\}_{n=1}^N \stackrel{\text{iid}}{\sim} p_{X,Y}$$



$$\text{minimize}_{f \in \mathcal{F}} \frac{1}{N} \sum_{n=1}^N \mathbf{1} [f(x_n) \neq y_n]$$





*a combinatorial graph*

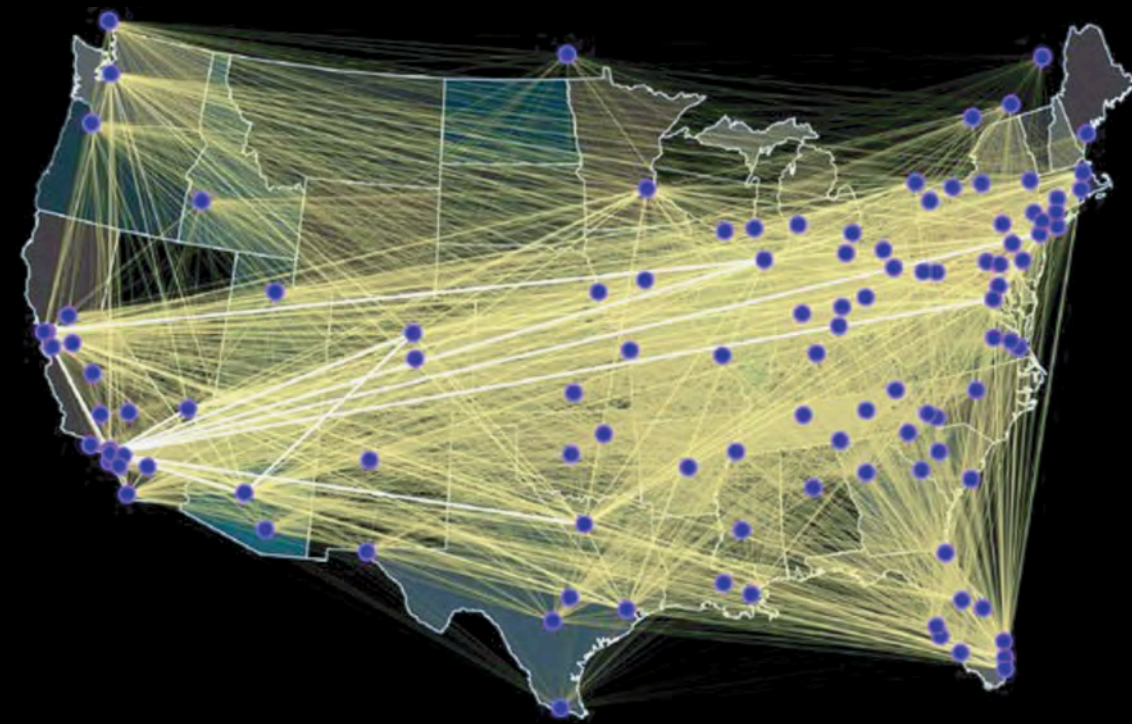
$$G = (V, E) \quad E \subseteq \binom{V}{2}$$

*an adjacency matrix*

$$A = (a_{ij})$$

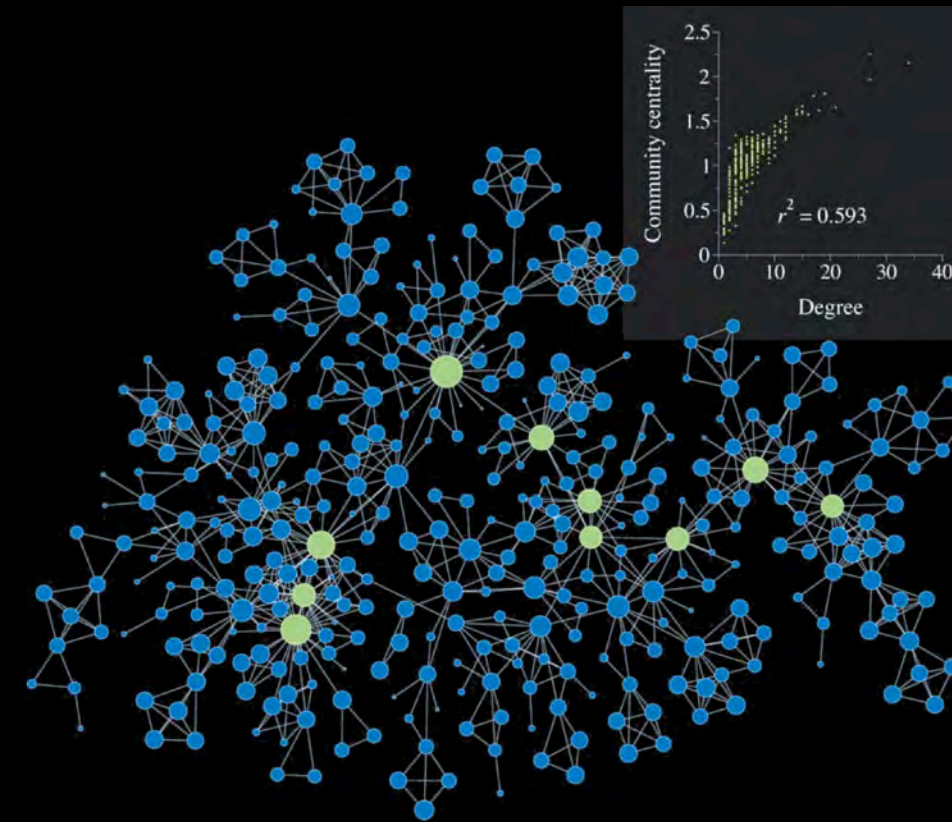
$$a_{ij} = \begin{cases} 1 & (i, j) \in E \\ 0 & (i, j) \notin E \end{cases}$$

Benson et al. 2016



*air transport network in the US*

Newman 2006

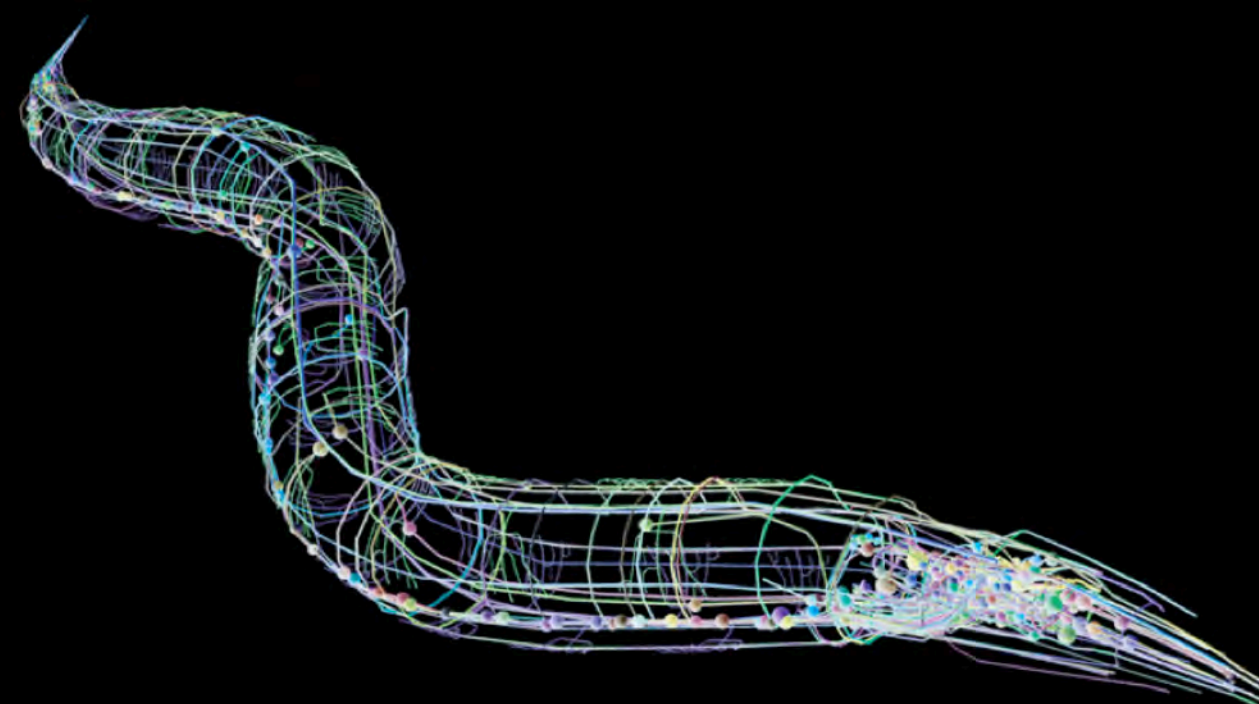


*collaboration network of network scientists*

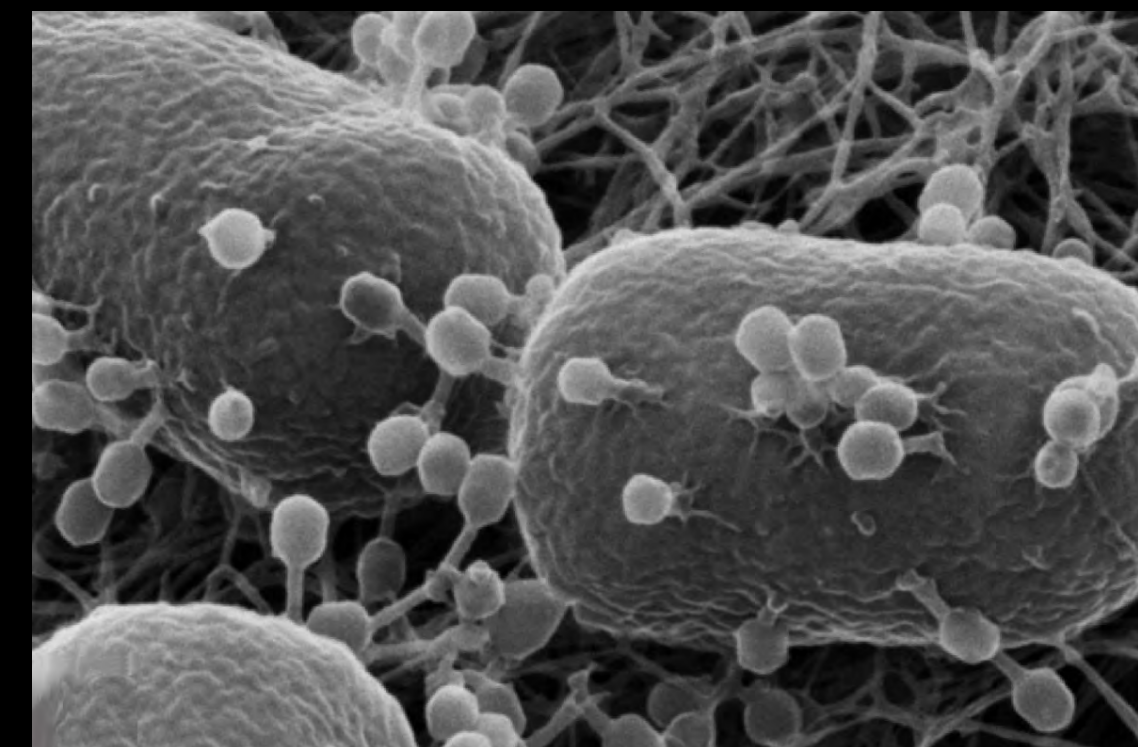
von Mering 2002



*protein-protein interaction graph in yeast*

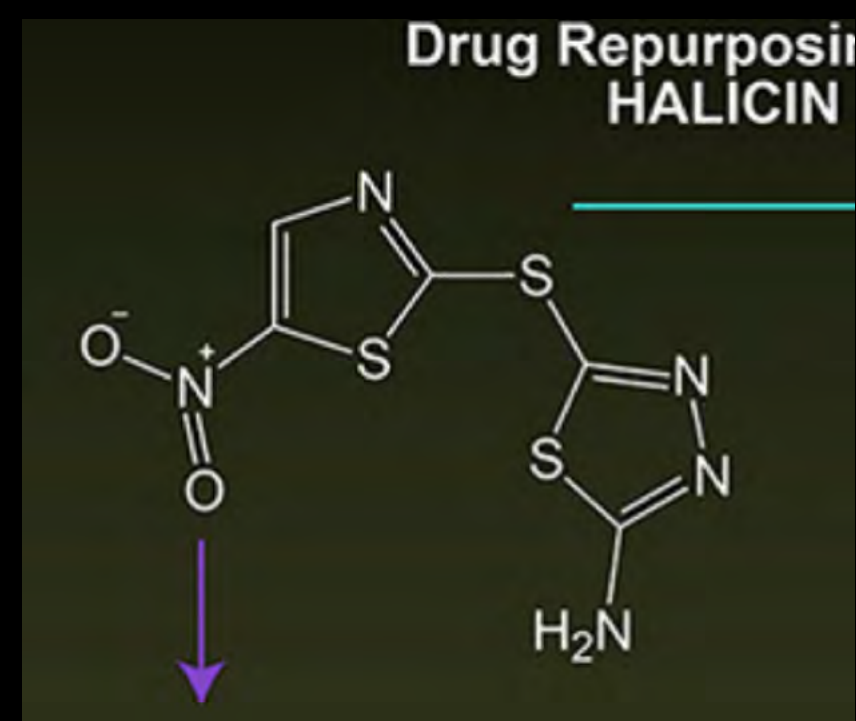


*neural network of c. elegans*

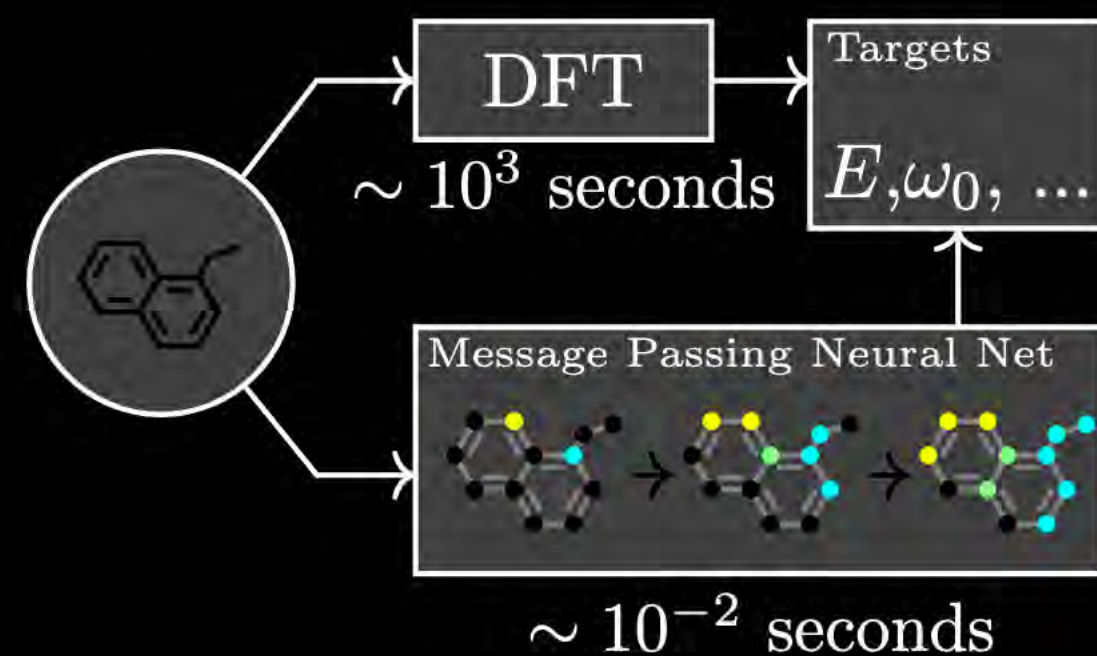


*reactions between metabolites in e. coli*

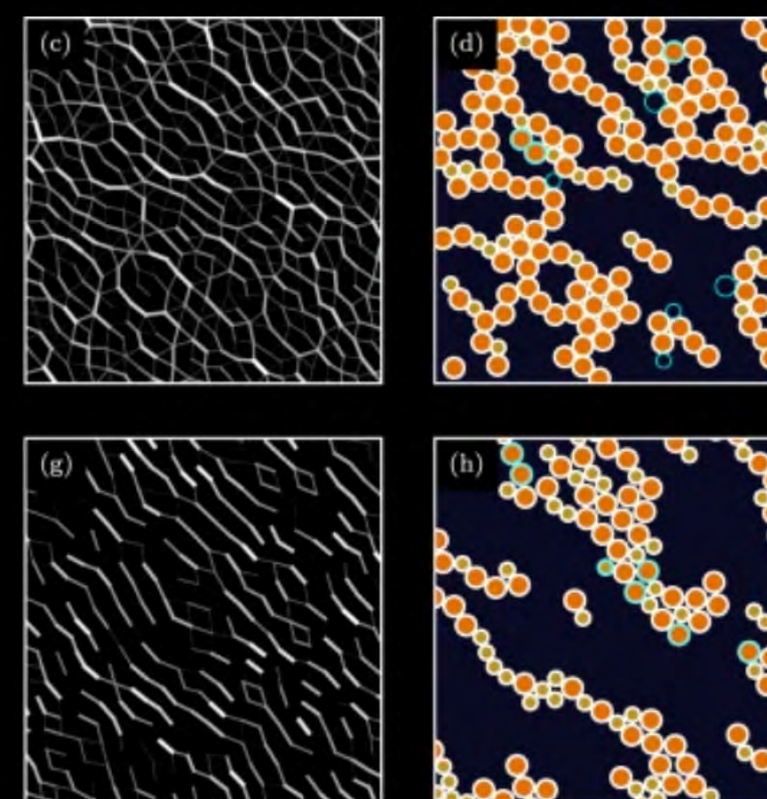
# killer applications



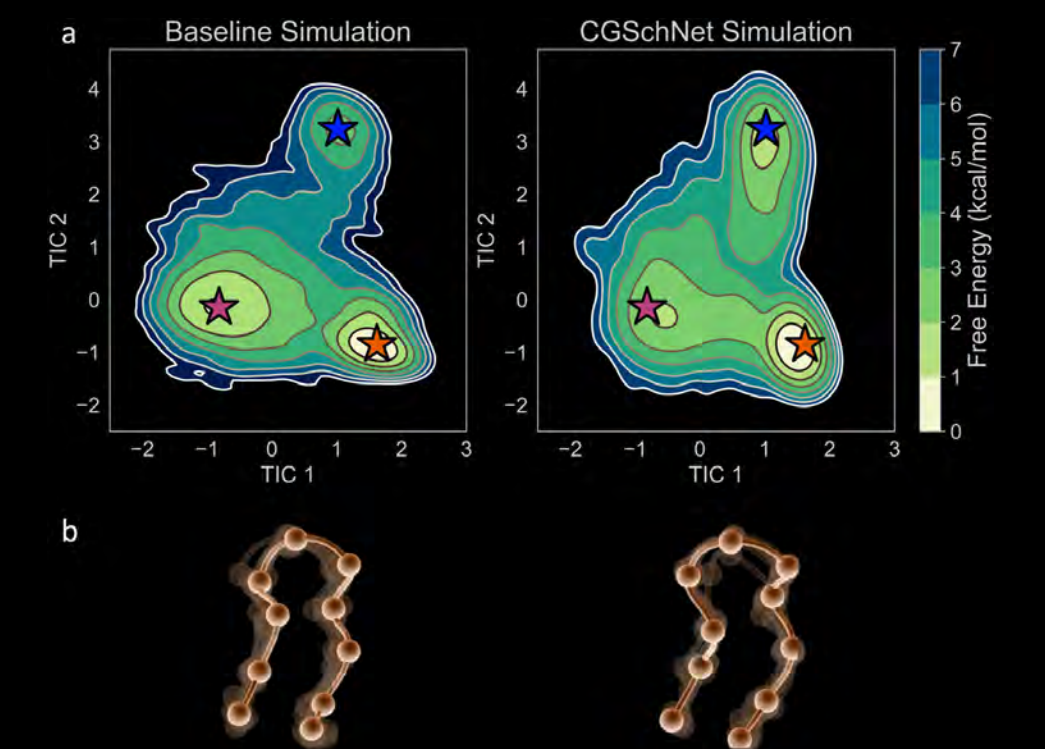
molecule design  
Stokes et al. 2020



quantum chemistry  
Gilmer et al. 2017



force chains in jammed solids  
Mandal, Caser, Sollich 2022

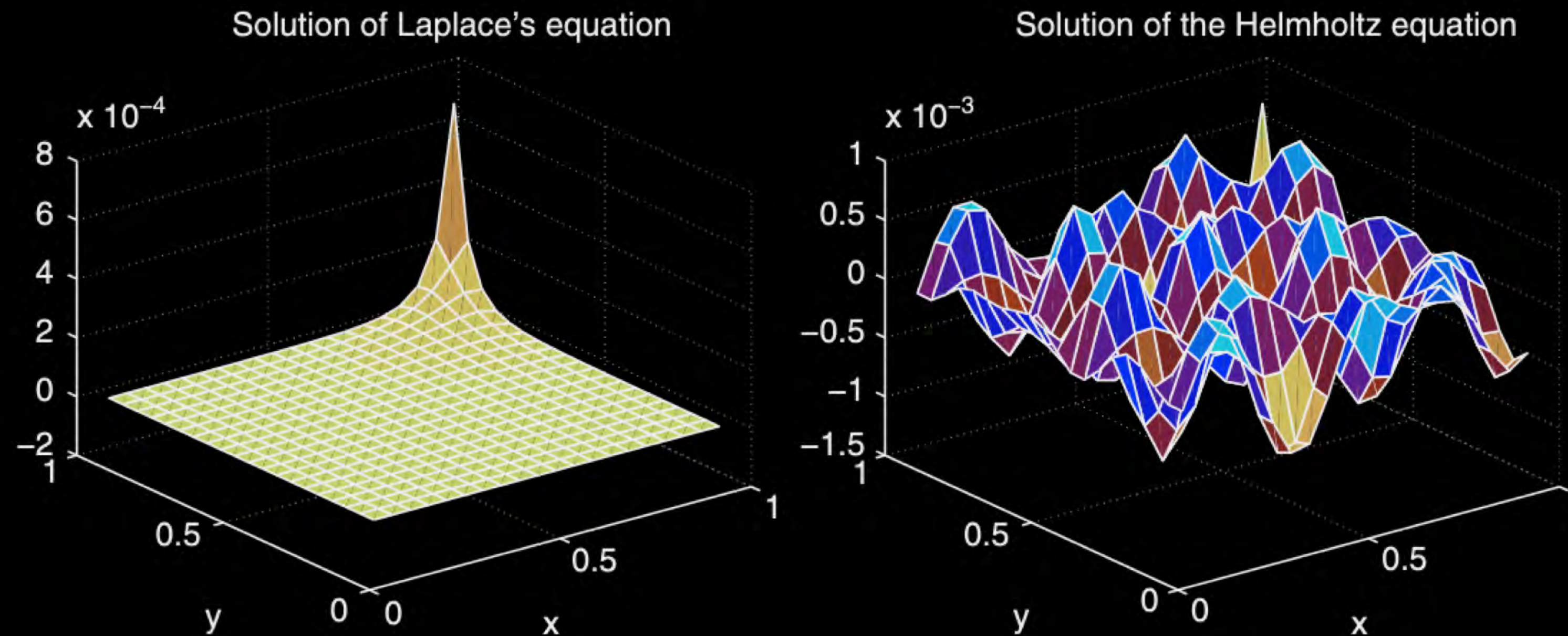


fast molecular dynamics  
Husic et al. 2020



$$-(\Delta - \eta)u = f \quad (\eta > 0) \qquad -(\Delta + k^2)u = f$$

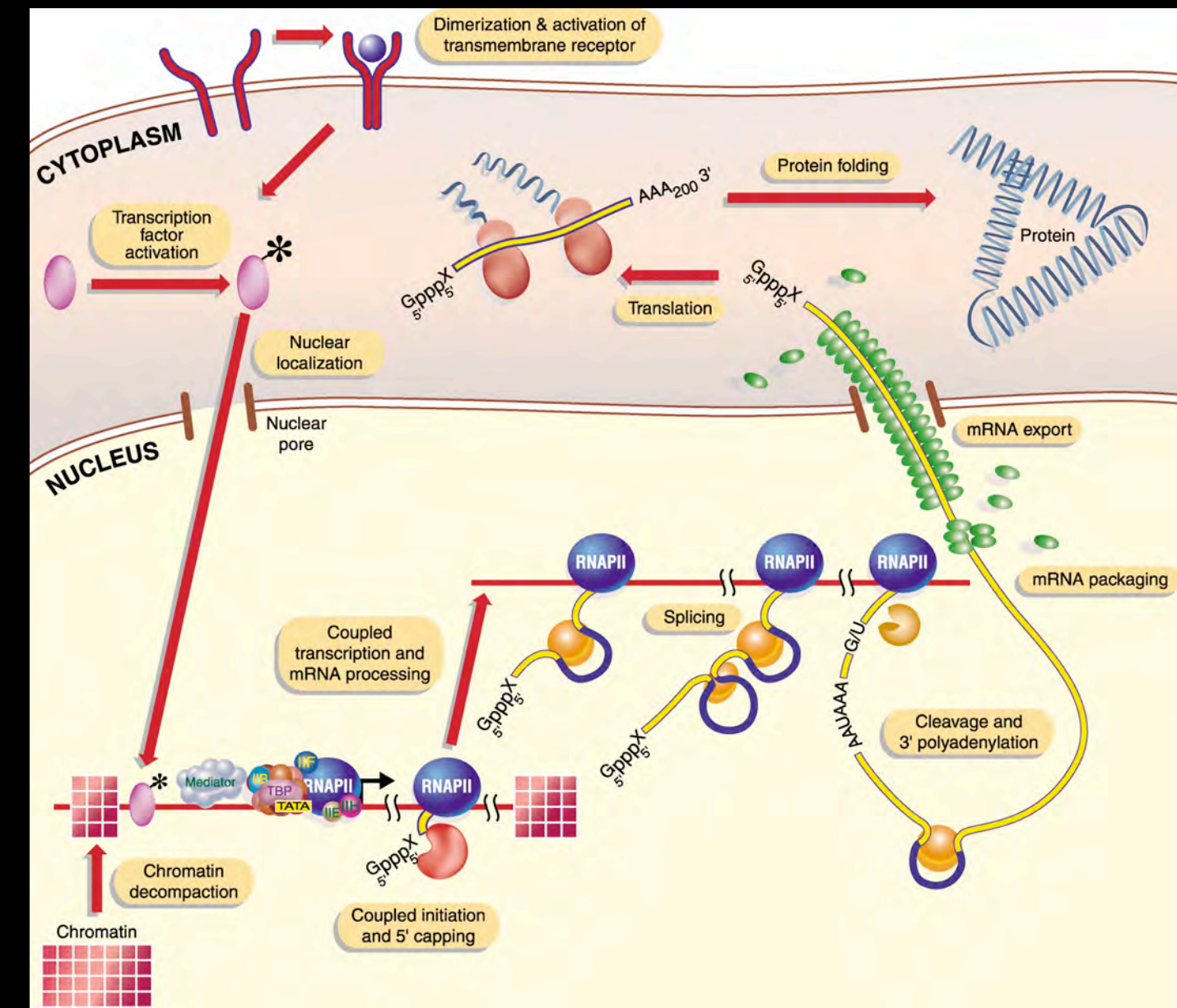
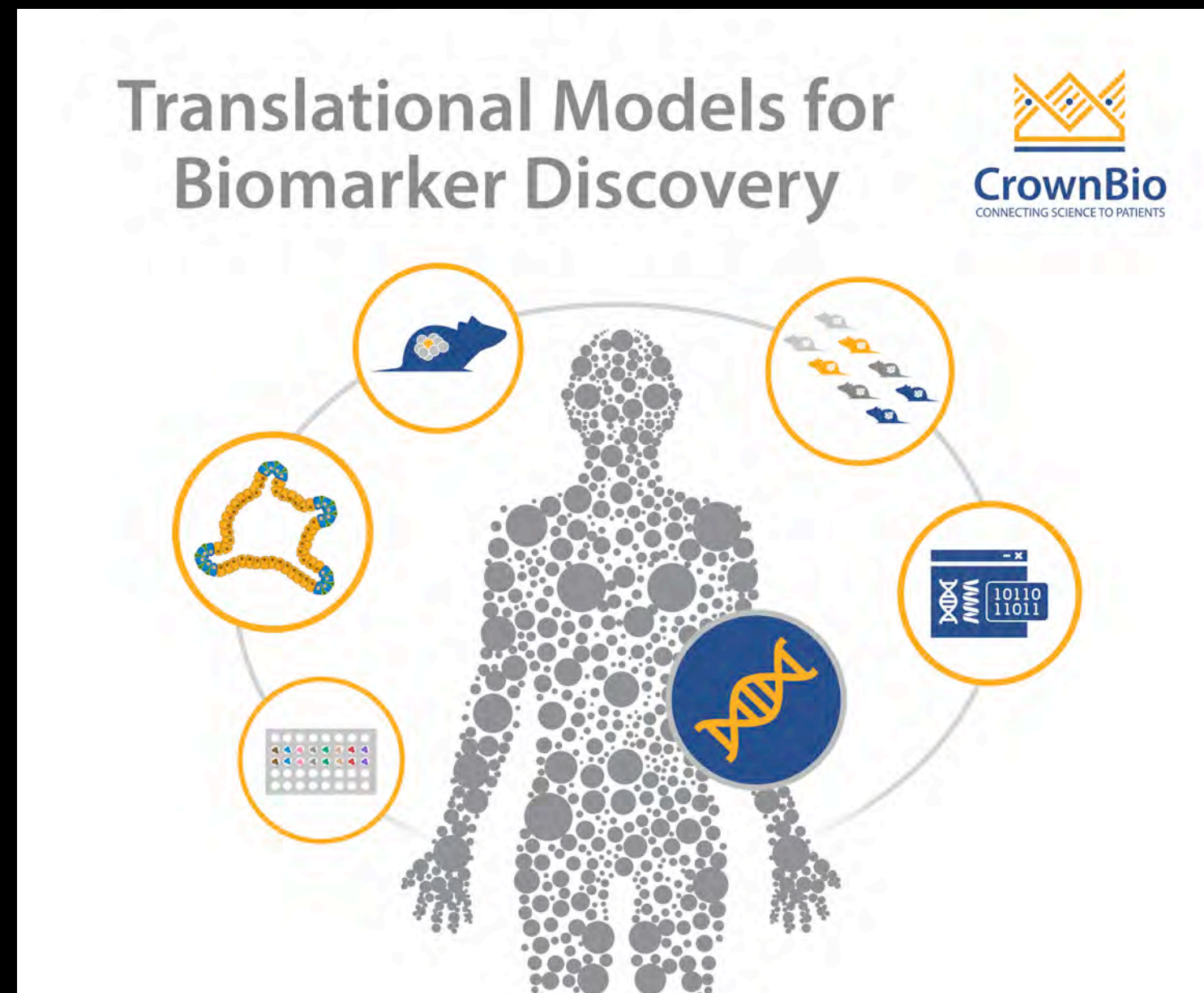
Ernst, Gander, 2012. **Why is it difficult to solve Helmholtz problems with classical iterative methods?**



**Fig. 1** Solution of Laplace's equation on the left, with a point source on the boundary, and on the right the solution of the Helmholtz equation, with the same boundary conditions

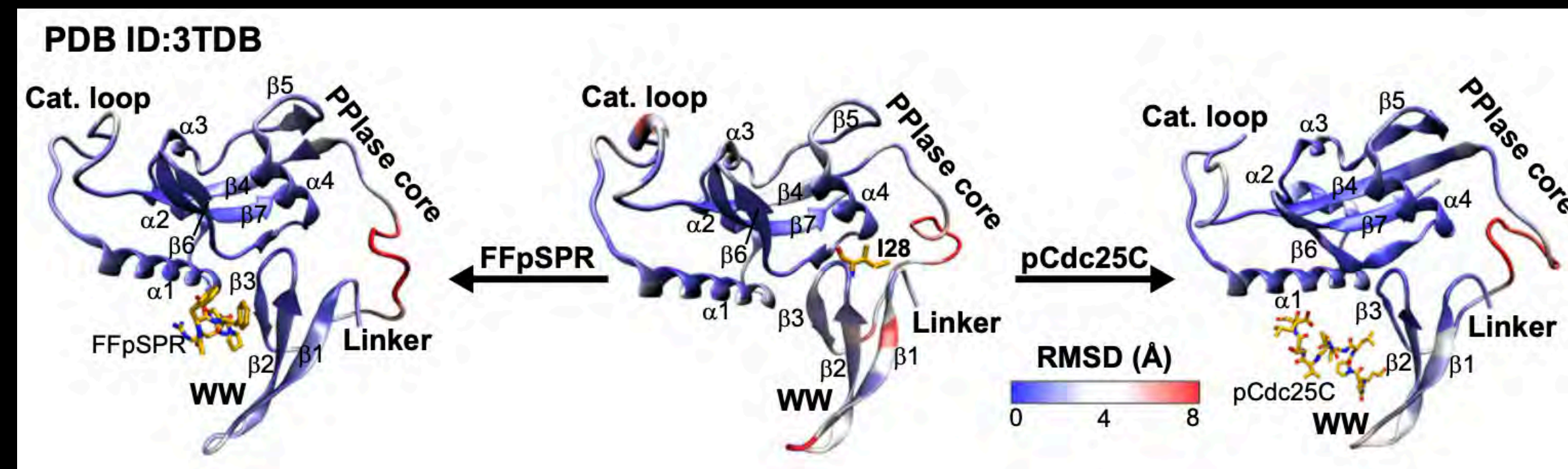
# link prediction in complex networks

applications of walkpooling we're hearing about

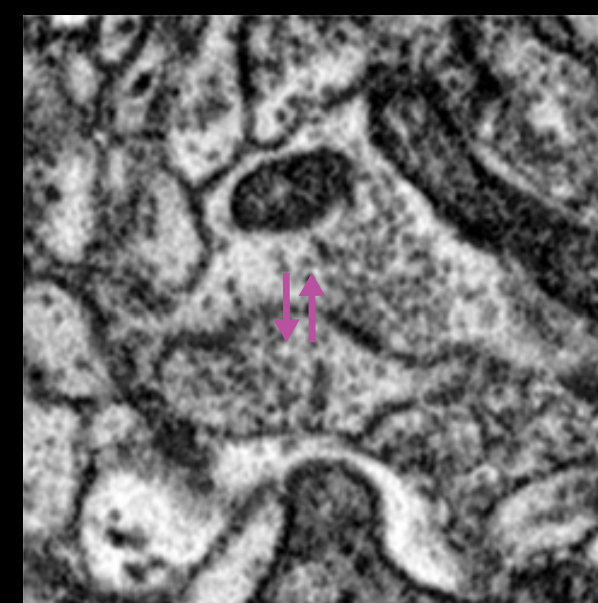


# learning dynamical systems on networks

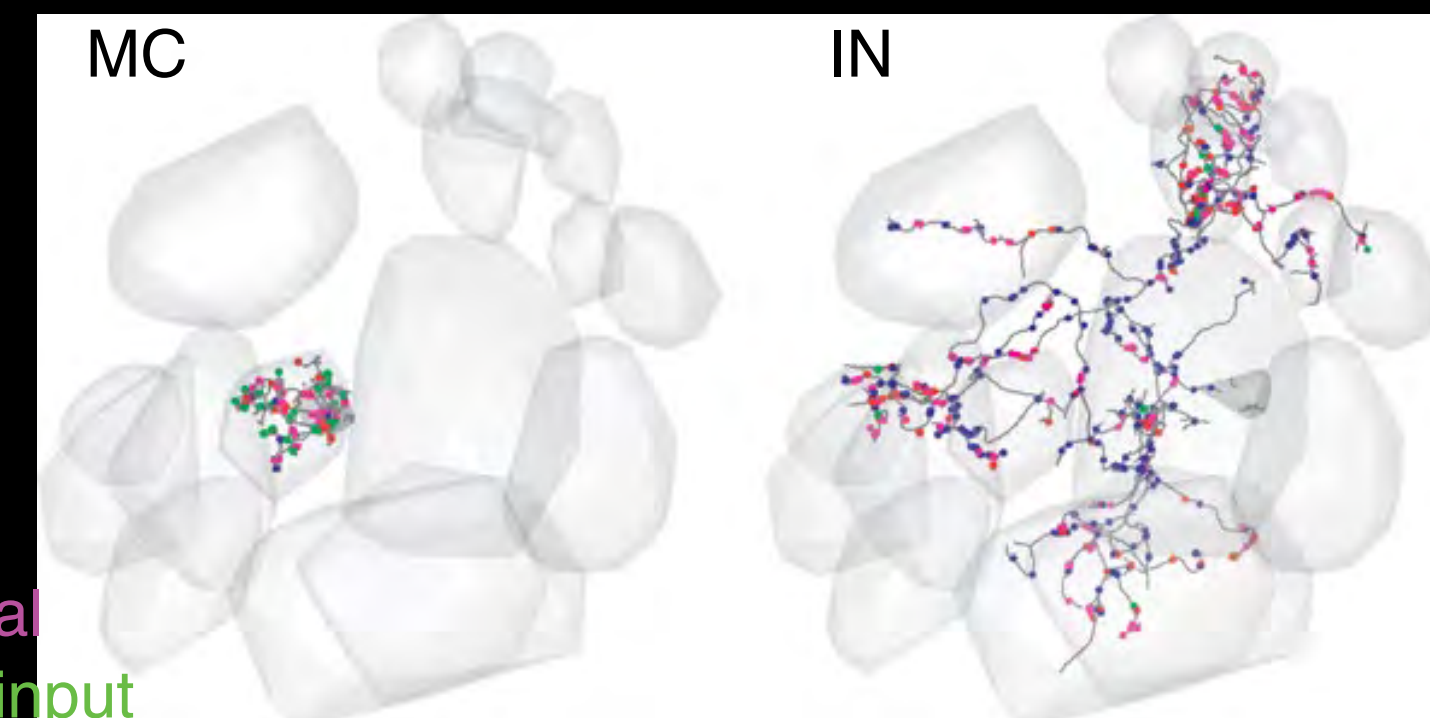
dynamics of proteins



dynamics of biological neural networks



Input  
Output  
Reciprocal  
Sensory input





# a few possible theses

modeling earthquake dynamics along the San Jacinto fault by neural point processes & reinforcement learning

applying FunkNN to model continuously deforming protein surfaces

using graph neural networks to model dynamics of granular material

applying transformers or diffusion models to ultrasound breast tomography

machine learning for exoplanet detection

theory for any of the above

## Be cautious

Machine learning is a **hot** topic

Many are interested in machine learning

Machine learning requires math

Not everyone is familiar with math

Make sure you are familiar with math

(credit Alex Schwing / UIUC)