Image Filtering (linear)

For more details see:
Digital Image Processing
by R.C. Gonzales & R.E. Woods

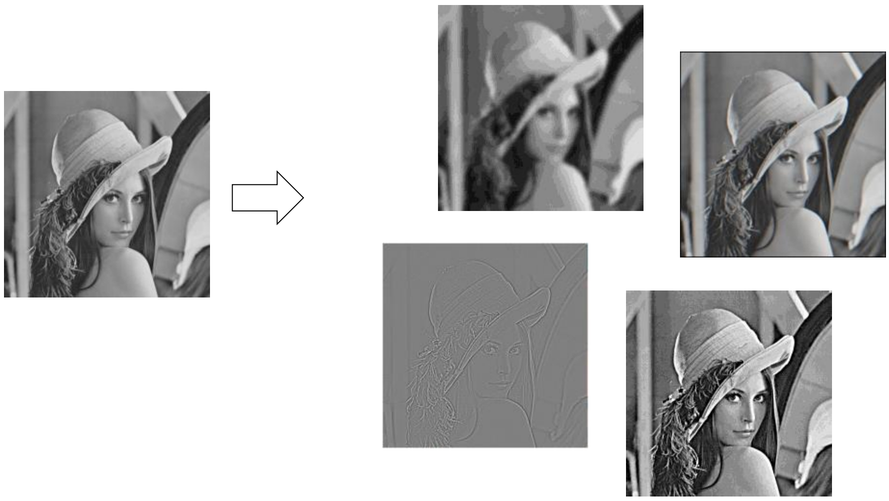


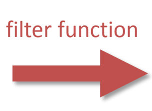
Image Filtering (linear)

- Each novel output pixel value O(x,y) is as linear function of the neighboring pixel values of I(x,y).
  The linear weights are stored in the filter kernel K(s,t) (also called filter or filter mask)

$$O[x,y] = \sum_{s=-a}^{a} \sum_{t=-b}^{b} k[s,t]\, I[x+s, y+t]$$

10	5	3
4	5	1
1	1	7

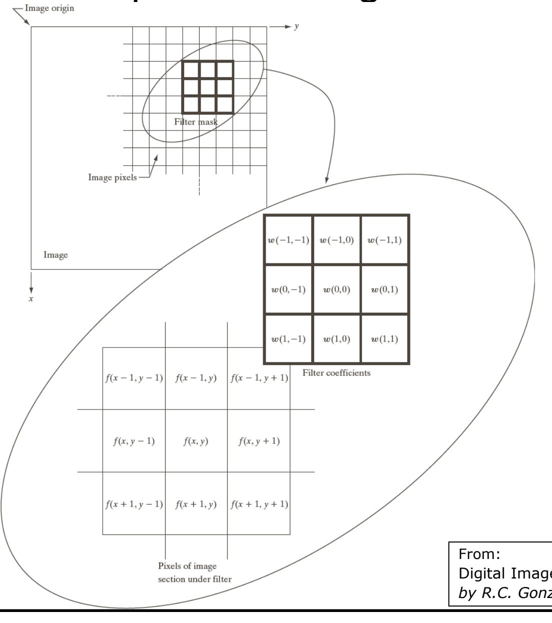
I Input Image



	7	

O Output image

Spatial Filtering



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Linear Filtering as correlation or convolution

- Cross-correlation: $O[x,y] = \sum_{s=-a}^{a} \sum_{t=-b}^{b} k[s,t]\, I[x+s, y+t]$

  Symbol: $O = k \otimes I$

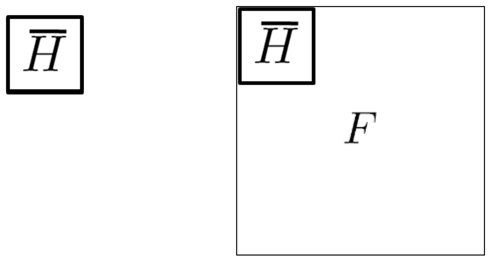
- Convolution: $O[x,y] = \sum_{s=-a}^{a} \sum_{t=-b}^{b} k[s,t]\, I[x-s, y-t]$

  Symbol: $O = k * I$

  Convolution is **commutative** and **associative**

For symmetric kernels there is no difference !!!

Convolution



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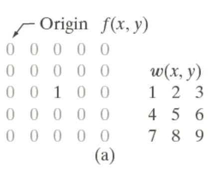

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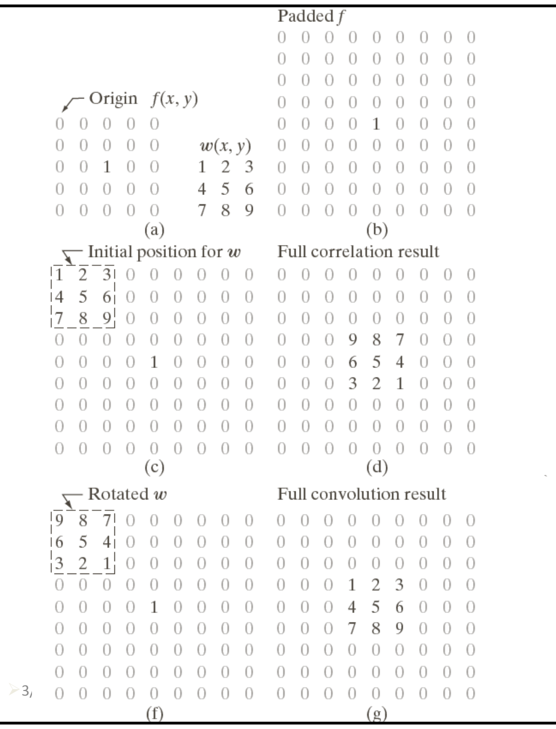

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Linear filters: examples

Original

*

0	0	0
0	1	0
0	0	0

=

Identical image

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Linear filters: examples

Original

$*$

0	0	0
1	0	0
0	0	0

$=$

Shifted left
By 1 pixel

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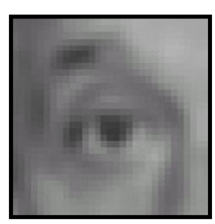
Linear filters: examples

Original

$* \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$=$



Blur (with a mean filter)

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Linear filters: examples

Original

$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

Sharpening filter
(accentuates edges)

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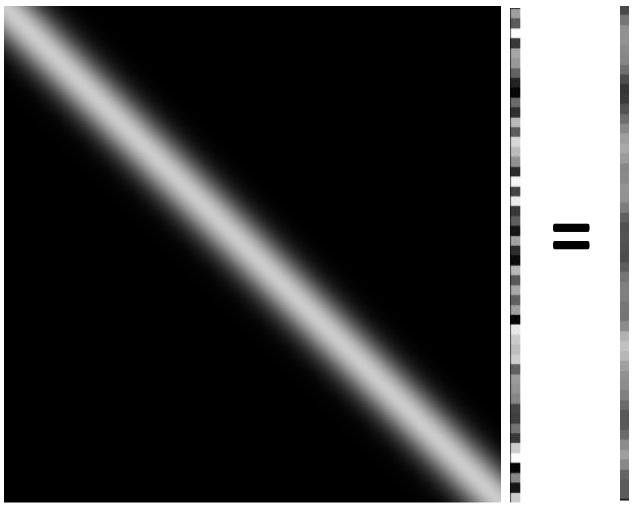
Separable Filter

An often used filter for blurring is the binominal mask.

Since this 2D mask can be separated into two 1D masks
the computational complexity can be heavily reduced!.

$$^{4}B = \frac{1}{256}\begin{pmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 6 \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 1 \end{pmatrix} \circ \frac{1}{16}\begin{pmatrix} 1 & 4 & 6 & 4 & 1 \end{pmatrix}$$

Filtering as matrix multiplication (1D)



Filtering as matrix multiplication (1D)

$$\begin{bmatrix}
0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 \\
0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 \\
0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\
0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2 & 0.2 \\
0.2 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0.2 & 0.2
\end{bmatrix}
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$