Seminar Bayesian Scientific Computing

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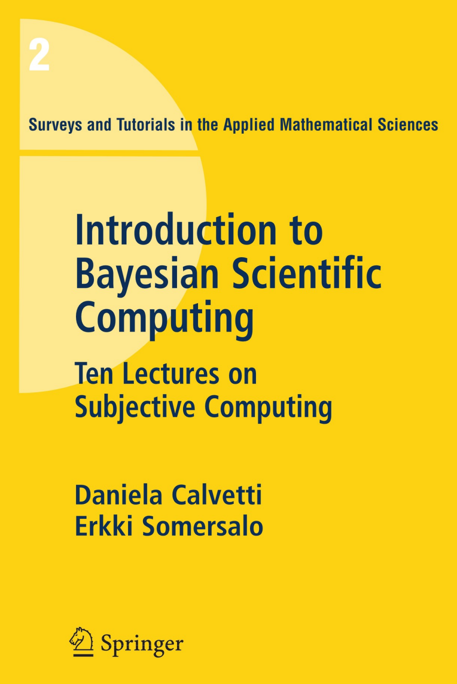
17th February 2020

Administrative Details

- Weekly presentations, approx 40-45 minutes.
- Every presenter has to write a summary (2-3 pages)
- Every **“opponent”** has to prepare 5 questions
  $\rightsquigarrow$ stating point for discussion.
- Mandatory rehearsal talks: one week before the presentation.
- Teaching material, presentations etc. on ADAM.
- Additional 3 credit points for programming project.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Surveys and Tutorials in the Applied Mathematical Sciences
Introduction to Bayesian Scientific Computing
Ten Lectures on Subjective Computing
Daniela Calvetti
Erkki Somersalo
Springer

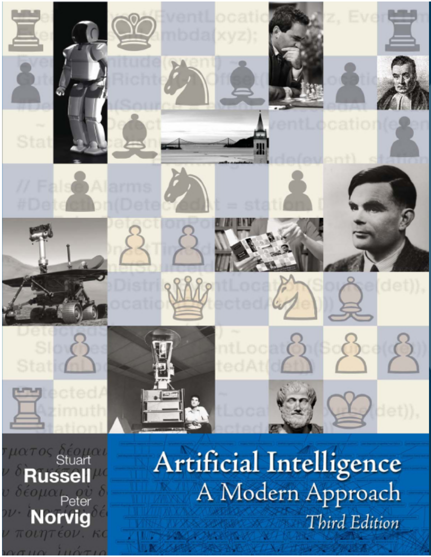
Timetable

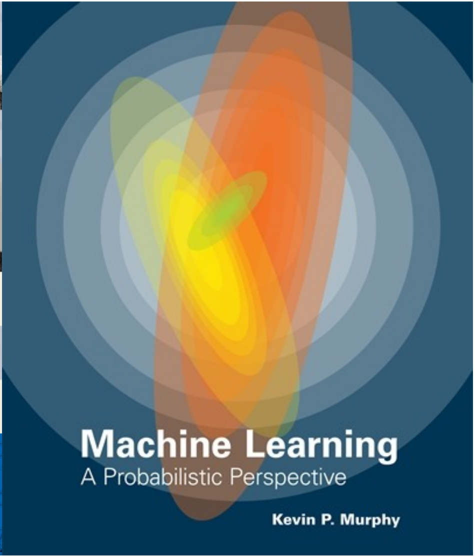
20.02. ...introduction
27.02. ...preparation phase
05.03. Fasnacht break
12.03. Chapter 1
19.03. Chapter 2
26.03. Chapter 3
02.04. Chapter 4
09.04. Easter break
16.04. Chapter 5
23.04. Chapter 6
30.04. Chapter 7
07.05. Chapter 8
14.05. Chapter 9
21.05. Chapter 10
28.05. Postponed presentations etc. (?)

Chapter 1: Inverse problems and subjective computing

- Inverse problems
- Statistical Inference
- Random variables / probability space /distribution / density etc.
- Bayesian /Frequentist interpretation of probabilities
- Expectation, variance, covariance & correlation
- Joint and conditional probabilities
- Multivariate Gaussian distribution
- Central Limit Theorem

Additional literature:
Chapter 13 in **Russell & Norvig**, Artificial Intelligence: A Modern Approach, Pearson.
Sections 2.1-2.6 in K. Murphy, Machine Learning: A Probabilistic Perspective, MIT press


Stuart
Russell
Peter
Norvig
Artificial Intelligence
A Modern Approach
Third Edition


Machine Learning
A Probabilistic Perspective
Kevin P. Murphy

Chapter 2: Basic Problem of Statistical Inference

- Statistical modeling and analysis
- Parametric and non-parametric approach
- Averaging - law of large numbers
- estimators: empirical mean and covariance
- credibility intervals / ellipses
- Maximum Likelihood estimation
- Multivariate Gaussian $\rightsquigarrow$ weighted least squares

Additional literature:
Sections 4.1, 4.3, 4.4 in **K. Murphy**, Machine Learning: A Probabilistic Perspective, MIT press

3: Ignorance- Randomness as a lack of information

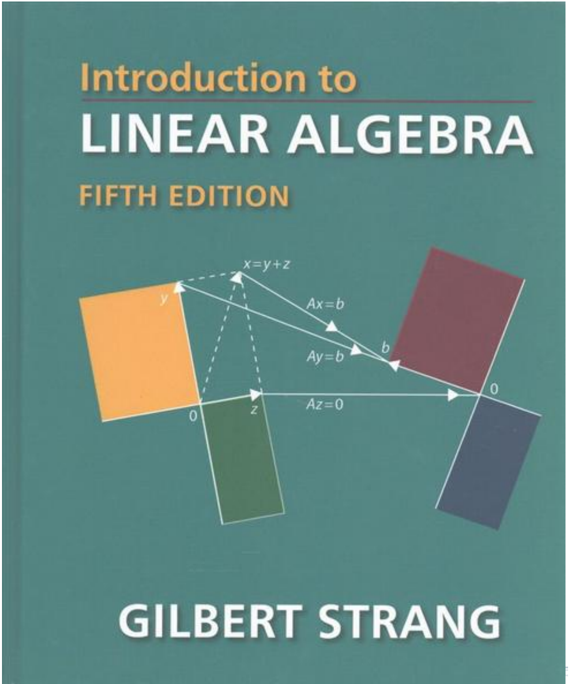
- Conditioning and marginalization
- Construction of the likelihood function
- Additive noise model
- Choice/construction of priors
- Empirical Bayes methods
- Posterior densities as solutions of statistical inverse problems
- Tikhonov regularization, MAP estimator

Additional literature: (general Bayesian statistics, empirical Bayes etc.)
Chapter 13 in **Russell & Norvig**, Artificial Intelligence: A Modern Approach, Pearson.
Chapter 5 in **K. Murphy**, Machine Learning: A Probabilistic Perspective, MIT press

4: Basic problem in numerical linear algebra

- Solution - undetermined, formally determined, over-determined
- Minimum norm / Least squares solutions
- Direct linear system solvers, Cholesky decomposition
- Conditioning
- Singular Value Decomposition
- Iterative solvers
- Fast Fourier Transform
- Krylov subspace, Conjugate gradient methods etc.
- Model selection - overregularization and underregularization

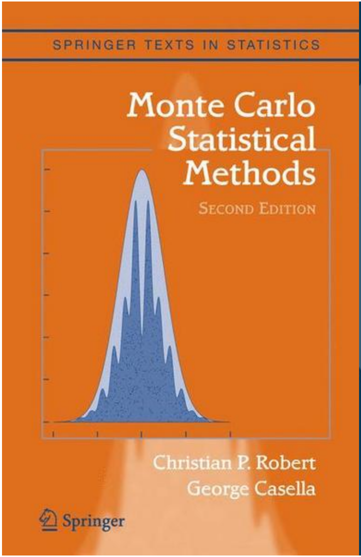
Additional literature: ...many (applied) LA books, for instance
Gilbert Strang, Introduction to Linear Algebra, Wellesley Cambridge Press.

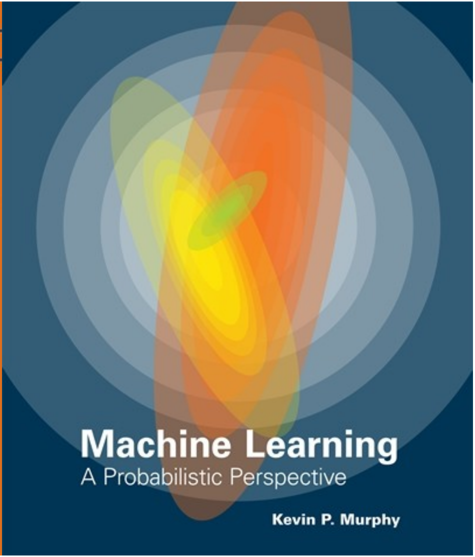

Introduction to
LINEAR ALGEBRA
FIFTH EDITION
x=y+z
y
Ax=b
Ay=b
b
0
z
Az=0
0
GILBERT STRANG

5: Sampling

- Monte Carlo Integration
- Random number generation (normal and uniform distribution)
- Inverse Cumulative distribution rule
- Rejection sampling
- Markov chain Monte Carlo, Metropolis Hasting Algorithm

Additional literature:
Section 2.7 in **K. Murphy**, Machine Learning: A Probabilistic Perspective, MIT press.
Chapter 2 in **Robert & Casella**. Monte Carlo Statistical Methods, Springer.


SPRINGER TEXTS IN STATISTICS
Monte Carlo Statistical Methods
SECOND EDITION
Christian P. Robert
George Casella
Springer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6: Statistically Inspired Preconditioners

- Prior conditioners - Conjugate gradients (CGLS), Generalized Minimal RESidual (GMRES), MAP
- Sample based preconditioners, PCA model reduction
- How to estimate a quantity x from observations b that are related via a computational model?
- Gaussian approximation
- Anomaly detection
- PCA applications

7: Conditional Gaussian densities and predictive envelopes

- Gaussian conditional densities
- Schur complements
- Interpolation, splines and conditional densities
- Predictive envelopes - extrapolation problems

Additional literature:
Sections 4.3 & 4.4 in **K. Murphy**, Machine Learning: A Probabilistic Perspective, MIT press

8: More applications of Gaussian conditioning

- Linear Gaussian models with additive noise
- Optimal (Wiener) filtering
- Boundary conditions
- Deconvolution problems
- Smoothness priors - Dirichlet, Aristotelian etc.

9: Sampling

- MCMC
- Discrete time stochastic processes - Markov processes
- Random walk
- Markov chains, invariant distribution
- Convergence: detailed balance
- Practical problems: burn-in phase, acceptance ratio, auto-correlation etc.

Additional literature:
Section 24 in **K. Murphy**, Machine Learning: A Probabilistic Perspective, MIT press
Chapters 6,7 in **Robert & Casella**, Monte Carlo Statistical Methods, Springer.

10: Hypermodels, dynamic priorconditioners and Bayesian learning

- Priors usually depend themselves on parameters: how to chose them?
- (Hierarchical) construction of priors
- Bayesian hypermodel: hyperprior, posterior etc.
- Inference: MAP or marginalization?
- Bayesian hypermodels and priorconditioners

Additional literature: (hierarchical Bayesian models etc.)
Chapter 5 in **K. Murphy**, Machine Learning: A Probabilistic Perspective, MIT press