

43075-01 Probabilistic Shape Modelling

Lecturers

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Introduction 25. February 2020

Deadline **17. March 2020** (Discussion / presentation during exercise session)

Exercise 0: Before you begin

0.1 Forming groups

You should work on the exercises and projects in this course in groups of two students. We will discuss your answers to the questions during the exercise session on the 17. March (14.15 - 18.00). *Please send us the names of the persons in your group via email to marcel.luethi@unibas.ch no later than 04. March. Once we have all the names, we will send a schedule with the timeslots for the discussions.*

1. Developing with Scalismo

1.1. Setting up the IDE

For the project, we will not work with ScalismoLab, but will use the latest version of Scalismo (the library).

To set up your development environment, follow the steps indicated in the guide *Using Scalismo in an IDE*, which you find here: <https://scalismo.org/tutorials/ide.html>.

1.2. Scalismo tutorials

Work through the tutorials 1 - 4 on <https://scalismo.org/tutorials>. Paste the code snippets into your own Scala project and run them from the IDE. Note down your questions. We can discuss them during the exercise session.

2. Femur project

2.1. Downloading data

To obtain the data for the first course project, you will need to create an account on the SMIR repository in order to be able to access the project data. To do this, follow the instructions in Step 4.11: "Femur Project: First Steps", section "Create your account on SMIR" on FutureLearn.

2.2. Rigid alignment

Once your SMIR account is created, you can proceed to downloading the femur data, by following the instructions in step 5.11. "Femur project: data download and preparation". The downloaded femur bones are not yet aligned. Perform a rigid alignment using the provided landmarks (see step 5.11 for details).

Save all the aligned meshes. You will need them in the second step of the femur project, which will be part of the next exercise sheet.

2.3. Generalized Procrustes Analysis

Implement the generalized Procrustes method and apply it to the points defined by the landmarks (which you can access by calling `lm.point` on a landmark `lm`). Visualize how the mean position of the landmarks change in each iteration. Note that to visualize a point cloud, you can call the `show` function with an object of `IndexedSeq[Point[_3D]]`.



Remark: The goal of this exercise is, besides learning about Procrustes analysis, also to get familiar with programming in Scala and Scalismo. Please use the forum on adam (https://adam.unibas.ch/goto_adam_frm_926319.html) in case you have questions regarding Scalismo/Scala.

3. Theory questions

3.1 Definition of shape

In the classical definition of shape, not only rotation and translation but also scaling is normalized.

- What can you say about the variability of different shapes that a model represents, when scaling is normalized? Is it larger or smaller than when you do not normalize scale? Explain.
- Assume we would also normalize with respect to a reflection along one of the coordinate axis? How is the variability of different shapes in the model affected in this case? Could you still detect differences in shapes of left and right hands?

3.2 Bivariate normal distribution

Given the following bi-variate normal distribution

$$N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 & 6 \\ 6 & 9 \end{pmatrix}\right).$$

Plot some samples from this normal distribution (using a tool of your choice, such as Matlab, Python, R, Scala, ...). What can you observe? Can you explain what is happening?

3.3 Rank of a statistical shape model

Assume we are given 10 example surfaces in 2D in correspondence and we let each surface be defined on 1024 points. Assume further, that we represent each example s^i as a vector $s^i = (s_x^1, s_y^1, \dots, s_x^{1024}, s_y^{1024})$ and estimate the covariance matrix using the formula

$$\Sigma = \frac{1}{n-1} \sum_{i=1}^n (s^i - \bar{s})(s^i - \bar{s})^T$$

where \bar{s} is the sample mean

$$\bar{s} = \frac{1}{n} \sum_{i=1}^n s^i$$

What is the rank of the covariance matrix? Explain why.

Hint: In case you are not sure how to start, just create 10 random vectors $s^i, i = 0, \dots, 10$, compute the covariance matrix and observe how the rank changes when you increase the dimensionality.

3.4 Gaussian processes and multivariate normal distribution

Given a discrete Gaussian process $GP(\mu, k)$, with mean function $\mu : \tilde{\Omega} \rightarrow \mathbb{R}^3$ and covariance function $\mu : \tilde{\Omega} \times \tilde{\Omega} \rightarrow \mathbb{R}^{3 \times 3}$ defined on some discrete domain $\tilde{\Omega}$.

Assume your task is to develop a IO method, which can store the GP in a file and load it again from file. Would it be sufficient to store the mean vector μ and covariance matrix and Σ , obtained by evaluating the mean and covariance functions at all the points of the domain $\tilde{\Omega}$? Why or why not?

