10907 Pattern Recognition

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Exercise 1 — Normal Distribution

Introduction	24.09
Deadline	30.09 (on paper, Spiegelgasse 1)

1 Multivariate Normal Distribution [3p]

Consider a bivariate normal population with $\mu_1 = -2, \mu_2 = 1, \sigma_1^2 = 6, \sigma_2^2 = 6$, and with cross correlation coefficient, $\rho_{12} = -\frac{1}{2}$.

1. Expand the full probability density [1p]
2. Determine the main axes and sketch the constant-density contour at one standard deviation [2p]

2 Independence [3p]

Consider $\boldsymbol{X} = [X_1, X_2, X_3]^\intercal$ distributed according to $\mathcal{N}(\boldsymbol{X} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma})$ with

$$\boldsymbol{\mu} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & -3 \\ 0 & -3 & 6 \end{bmatrix}.$$

Which of the following pairs of random variables are independent? Explain.

1. X_3 and X_1
2. X_3 and X_2
3. $2X_1 - X_2 - X_3$ and $X_3 - X_2$

3 Conditional Distribution [2p]

Calculate the conditional distribution of X_1, given that $X_2 = x_2$ in the joint distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Compare the conditional distribution $P(X_1 \mid X_2 = 1)$ to the marginal distribution $P(X_1)$ in a plot.

$$\boldsymbol{\mu} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \qquad \boldsymbol{\Sigma} = \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

4 Classification [2p]

Classify a point $\mathbf{X} = [-2.0, -1.8]$ into one of two classes, where each class follows a normal distribution with parameters $\boldsymbol{\mu_1} = [-4, -2]$ and $\boldsymbol{\mu_2} = [-1, -2]$ and

(a) isotropic and identical covariance matrices.

(b) covariance matrices:

$$\boldsymbol{\Sigma_1} = \begin{bmatrix} 1.5 & 1.8 \\ 1.8 & 6 \end{bmatrix}, \qquad \boldsymbol{\Sigma_2} = \begin{bmatrix} 1.5 & 0.9 \\ 0.9 & 0.6 \end{bmatrix}.$$