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Graduate Student Seminar

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Seminarraum 00.003, Spiegelgasse 1

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The maximal number of lines going through the same point on del Pezzo surfaces of degree one

Abstract

Del Pezzo surfaces are surfaces that can be obtained from the plane by a construction called blow-up. They can be classifed by their degree, and we know exactly how many lines are contained in these surfaces for each degree over an algebraically closed field. Famous examples are smooth cubic surfaces in the space $\mathbb{P}^3$, which are del Pezzo surfaces of degree three. These contain 27 lines, of which at most three can go through the same point. Similarly, a del Pezzo surface of degree two contains 56 lines, of which at most four can go through the same point. In both of these cases, this maximum is given by the incidence graph of the lines in the surface.

In this talk I will look at del Pezzo surfaces of degree one, which contain 240 lines. I will show how we can find the lines on a del Pezzo surface and the corresponding incidence graph. I will then show the relation between the 240 lines on a del Pezzo surface of degree one and the classical root system E_8. Finally, I will explain how, using this and some classical geometry, we can show that in most cases the number of lines through one point is less than the bound given by the incidence graph. This is joint work with Ronald van Luijk.