Bernoullis Tafelrunde

Graduate Student Seminar

Monday, April 25 2022, 12:15-13:00
Hybrid seminar
Seminar room 05.001, Spiegelgasse 5 / Zoom

Johan Ulander

CHALMERS, Gothenburg

A positivity-preserving numerical scheme for stochastic heat equations with multiplicative space-time white noise

Abstract

The stochastic heat equation is a stochastic modification of the classical heat equation where we add a random forcing term. We say that a numerical scheme is positivity-preserving if the scheme produces only positive values whenever the initial state is positive. The goal of this research project is to propose a positivity-preserving numerical scheme for two types of stochastic heat equations with multiplicative space-time white noise as random forcing terms. It was established by C. Mueller in [1] that if the initial state of the considered stochastic heat equations is positive then the exact solution remains positive for all later times. This, however, does not imply that any numerical scheme that aims to approximate the solutions are also positivity-preserving. The goal of this research project/or talk is to propose and analyse a positivity-preserving numerical scheme for the stochastic heat equation with multiplicative space-time white noise.

I will start the talk by introducing the needed notions and notations and then discuss the theoretical properties of the exact solution to the stochastic heat equation. Then I will present a positivity-preserving Lie–Trotter splitting scheme and a convergence result. The scheme converges to the exact solution with order $\sqrt{\tau}$ in time, where τ is the time step size, and with order h^α in space, where h is the space step size, for any $\alpha \in (0, 1/2)$. The former order of convergence is novel and the latter order of convergence was established by Gyöngy in [2]. I will also outline the idea of how one can prove this result. This is a joint work with Charles-Edouard Brehier (University of Lyon) and David Cohen (Chalmers and University of Gothenburg).

[1] C. Mueller (1991). *On the support of solutions to the heat equation with noise.* Stochastics and Stochastic Reports, 37:4, 225-245.

[2] I. Gyöngy (1998). *Lattice approximations for stochastic quasi-linear parabolic partial differential equations driven by space-time white noise. I.* Potential Anal., 9, 1-25.