

BERNOULLIS TAFELRUNDE

GRADUATE STUDENT SEMINAR

Thursday, 22 December 2016, 12:15-13:00
Seminarraum 00.003, Spiegelgasse 1

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Integral points on polygons and modular forms

ABSTRACT

Given a convex polygon P in the plane one may ask for the number of points with integral coordinates on its boundary. If P has integral vertices and contains exactly one interior point, the answer is quite surprising: the sum of the numbers of integral points on the boundaries of P and its so-called dual polygon P^* is always 12. This fact can be proven by exhaustion, but this does not explain the appearance of the number 12. I will present a more sophisticated proof by Poonen and Rodriguez-Villegas which uses modular forms. In particular, we will see that the number 12 appears as the weight of Ramanujan's Delta function, which is the unique normalized cusp form of weight 12 for the full modular group.