Bernoullis Tafelrunde

Graduate Student Seminar

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Seminarraum 05.002, Spiegelgasse 5

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A gentle introduction to the Schinzel-Zassenhaus conjecture and its proof

Abstract

An *algebraic integer* is a complex number $\alpha \in \mathbb{C}$ which is a root of a monic irreducible polynomial $P \in \mathbb{Z}[X]$. The complete set of zeros of such a polynomial is a *conjugate set of algebraic numbers*, and the distribution of sets like this is an interesting topic in Number Theory. In particular, bounding the maximum absolute value of elements in these sets from below has been studied intensively over the years.

In 1965, Schinzel and Zassenhaus were not able to prove the following:

Conjecture (Schinzel-Zassenhaus Conjecture)**.** *There exists an absolute constant $C > 0$ such that for every non-cyclotomic monic irreducible polynomial $P \in \mathbb{Z}[X]$ of degree d with roots $\alpha_1, \dots, \alpha_d \in \mathbb{C}$, the following inequality holds:*

$$\max_{1 \leq i \leq d} |\alpha_i| > 1 + \frac{C}{d}.$$

The above conjecture was proved in 2019 by Dimitrov. In this talk I will introduce the relevant notions and sketch Dimitrov's proof.