Bernoullis Tafelrunde

Graduate Student Seminar

Monday, 12 December 2022, 12:15-13:00
Seminarraum 05.002, Spiegelgasse 5

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Introduction to real del Pezzo surfaces

Abstract

Del Pezzo surfaces are algebraic surfaces playing an important role in the classification of algebraic projective surfaces up to birational transformations. Over $\mathbb{C}$, a smooth del Pezzo surface of degree d is either isomorphic to $\mathbb{P}^2$ ($d = 9$) or to $\mathbb{P}^1 \times \mathbb{P}^1$ ($d = 8$) or to the blow-up of $\mathbb{P}^2$ in $1 \leq r \leq 8$ points in general position, where $d = 9 - r$.

Before explaining a bit that terminology, we will first see how algebraic projective varieties defined over a perfect field (e.g. $\mathbb{R}$ later on) naturally arise while studying some fibrations over $\mathbb{P}^1$, motivating the interest in the classification of smooth (rational) del Pezzo surfaces over a perfect field. We will then introduce the notions of real structures and real forms of complex projective surfaces (to enable us to use as many techniques coming from smooth complex projective surfaces as possible) and we will see that there are exactly two non-equivalent rational real models of $\mathbb{P}^1 \times \mathbb{P}^1$, namely the quadric surfaces given by $x^2 + y^2 - z^2 - w^2 = 0$ and $x^2 + y^2 + z^2 - w^2 = 0$ in $\mathbb{P}^3$. Finally, we will give the classification of smooth real del Pezzo surfaces of degree 6, that were initially introduced over $\mathbb{C}$ to be the blow-up of $\mathbb{P}^2$ in three points in general position (which is also isomorphic to the blow-up of $\mathbb{P}^1 \times \mathbb{P}^1$ in two points).