Bernoullis Tafelrunde

Math Students and PhDs Seminar

Thursday, 20.03.2025
Seminarraum 05.001

Remo von Rickenbach

Universität Basel

Bernoulli Explains Nonlinear Approximation

Abstract

Many problems from physics or engineering result in partial differential equations. In both cases, the unknown function u can, in most practically relevant applications, only be approximated numerically. Therefore, it is essential to use efficient algorithms to approximate u at as low costs as possible.

To build efficient algorithms, one first needs to understand how well a function of a given regularity can be approximated. For example, by using finite elements of grid size h on the unit interval I, it is well established that the approximation error decays as

$$\inf_{v_h \in V_h} \|u - v_h\|_{L^2(I)} \leq C h^s \|u\|_{H^s(I)}, \quad 0 \leq s \leq d,$$

where $d \in \mathbb{N}$ is the polynomial order of the trial functions involved.

However, what can we say about the approximation order if u is only piecewise regular, but admits a singularity and is therefore not in $H^s(I)$? For this, adaptive schemes, which rely on the concept of *nonlinear approximation*, need to be studied.

In this talk, some basic concepts and examples of approximation theory will be introduced and we will characterise the approximation spaces with respect to some common basis systems. In particular, we will see that the approximation spaces with respect to nonlinear approximation by wavelet bases contain Besov spaces which are strictly larger than the corresponding, classical finite element approximation spaces, showing that adaptive schemes strictly outperform classical schemes in the case of limited regularity.