Universität Basel

Departement
Mathematik und Informatik

Masterstudium Mathematik

im HS 2025 / FS 2026

Einleitung Prof. Dr. Enno Lenzmann

Universität Basel

Departement
Mathematik und Informatik

Vertiefungsmodule

- Numerik

- Stochastik

- Algebra - Geometrie - Zahlentheorie

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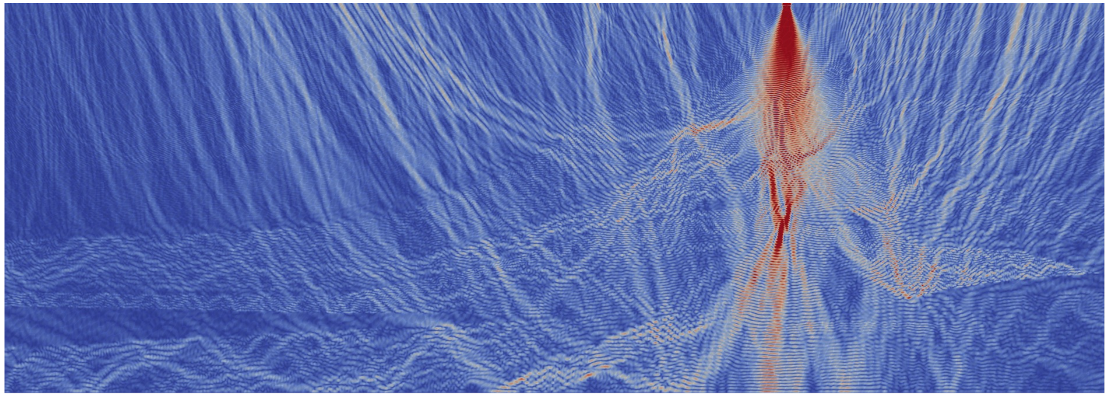
Numerical methods for partial differential equations

Prof. Dr. Marcus Grote

- **HS 2025: Numerical methods for PDE's I:**
Elliptic and parabolic equations (4 + 4 KP)

- **FS 2026: Numerical methods for PDE's II:**
Numerical methods for wave propagation (4 + 4 KP)

- **HS 2025 und F2026: Project (optional):**
Matlab + 3-5 p. report (2 KP)



Numerical methods for PDE’s I:

Elliptic and parabolic equations

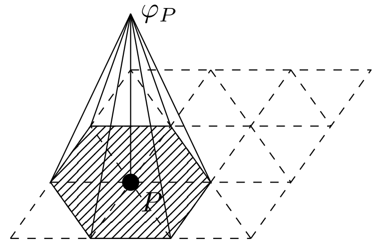
1. Classification of (linear) PDE’s, weak formulation, review of key results from functional analysis & Sobolev spaces

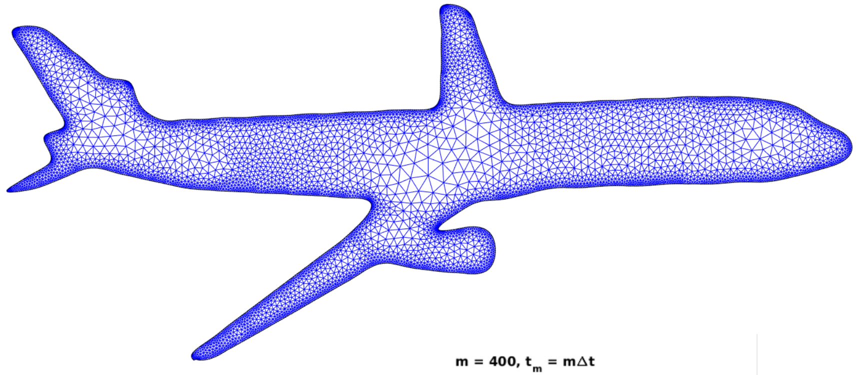
2. Finite Element Methods (FEM) for the numerical solution of elliptic PDE’s: 2D-Matlab implementation and convergence theory

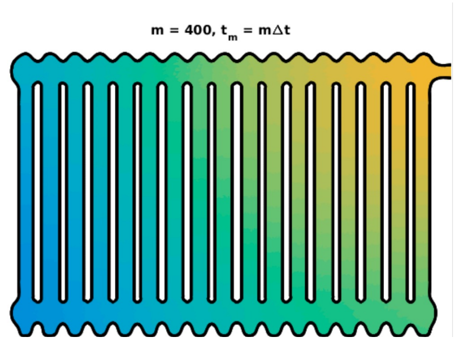
3. A posteriori error estimators, adaptivity, eigenvalue problems, discontinuous Galerkin (DG) methods

4. Variational crimes: curved boundaries, QF

5. Finite Element Methods for the numerical solution of parabolic PDE’s







Numerical methods for PDE’s II:

Numerical methods for wave propagation

1. Derivation and exact solutions of the acoustic, electromagnetic and elastic wave equations, frequency- vs. time-domain, method of characteristics

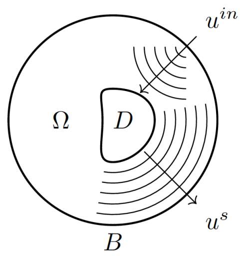
2. Finite Element Methods (FEM) for the numerical solution of the Helmholtz equation

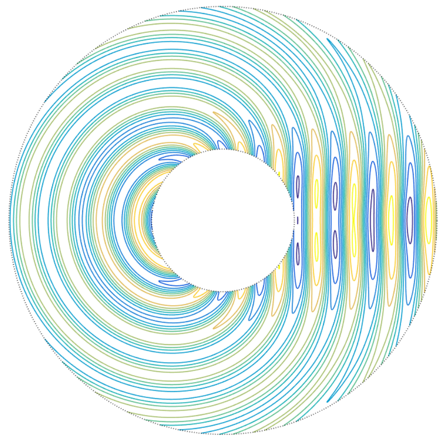
3. Unbounded domains and the Dirichlet-to-Neumann (DtN) map

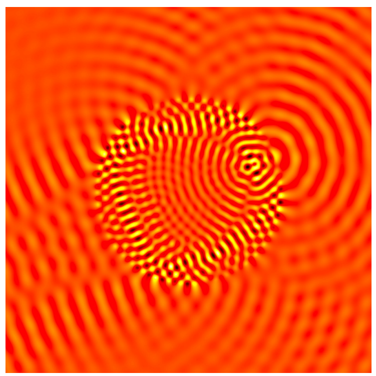
4. Finite Difference Methods (FDM) for the numerical solution of first- and second-order hyperbolic equations

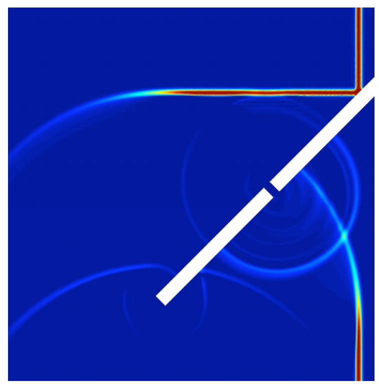
5. Finite Element Methods (FEM) for the numerical solution of the wave equation

6. Absorbing boundary conditions, Perfectly matched layers (PML)









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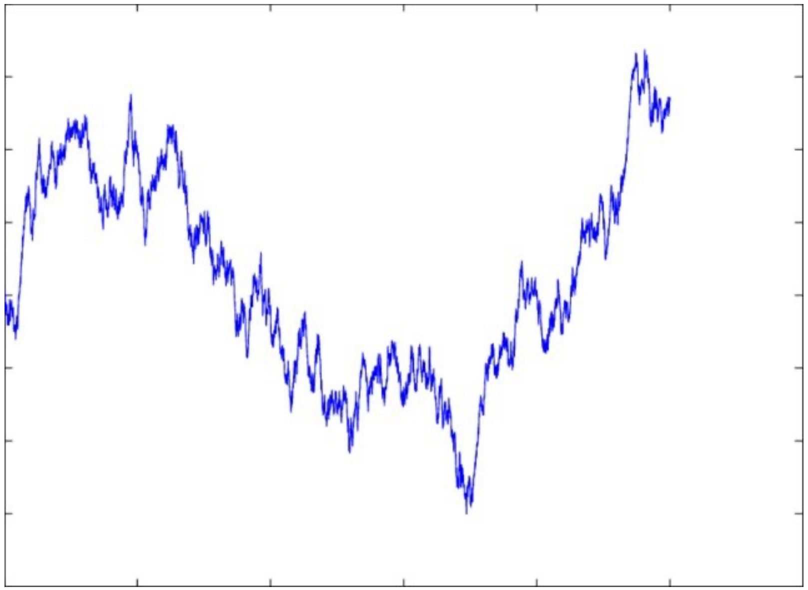
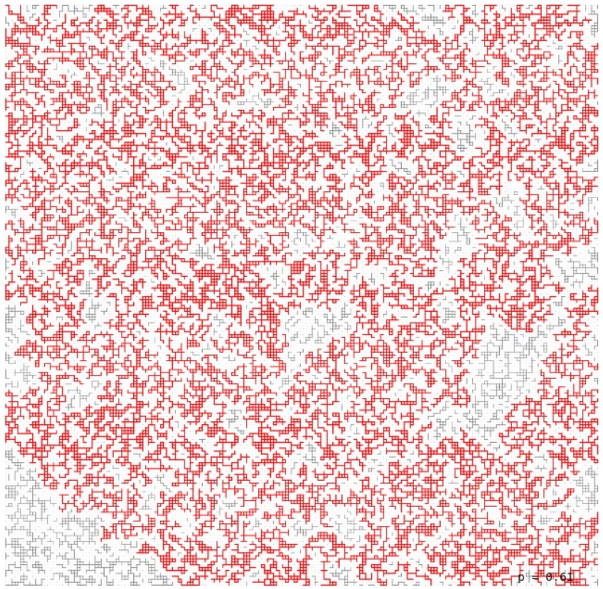
Stochastic Analysis (HS25) Selected topics in probability theory (FS26)

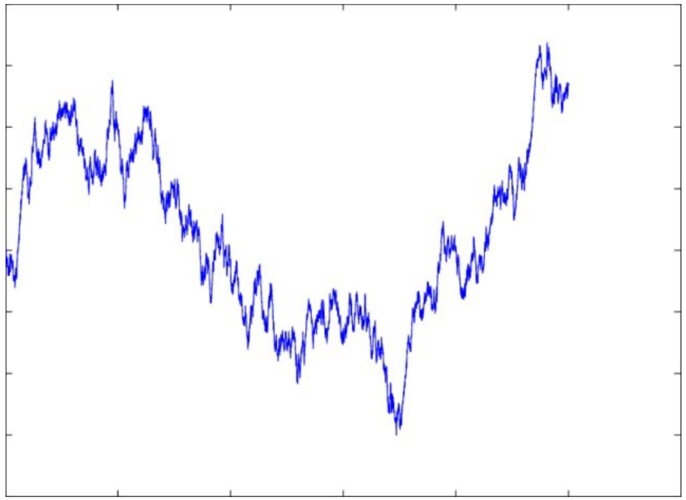
Prof. Dr. Jiří Černý





Stochastic Analysis (HS 2025)

Goal: Introduction to the theory of stochastic processes in continuous time.



Content:

- Brownian motion
- Martingales, Markov processes,
  Lévy processes, Gaussian processes
- Itō Integral
- Stochastic differential equations

$$dX_t = b(t, X_t)dt + \sigma(t, X_t)dB_t$$

- Connections to PDE
- Applications

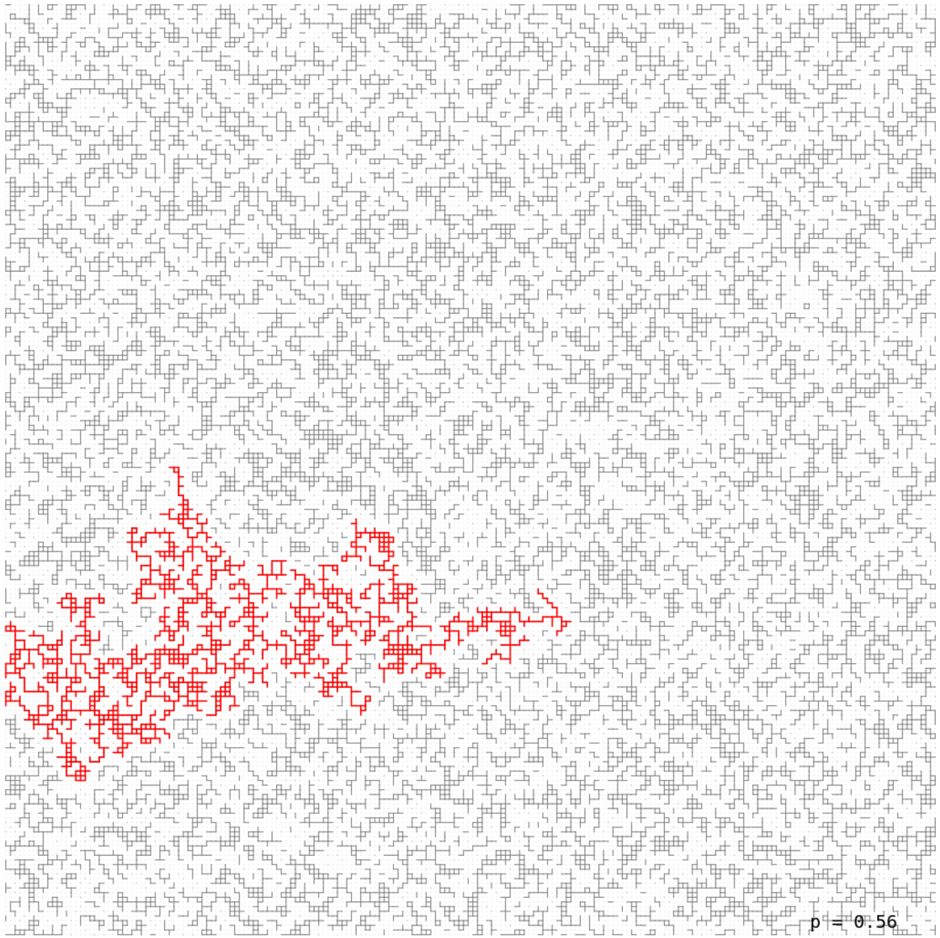
Literature: J.-F. Le Gall: Brownian Motion, Martingales and Stochastic Calculus

Selected topics in probability (FS 2026): Probability on graphs and networks

Goal: Theory of random graphs and networks and associated stochastic processes.

Content:

- Random graphs
- Percolation theory
- Random walks on random graphs and in random environment



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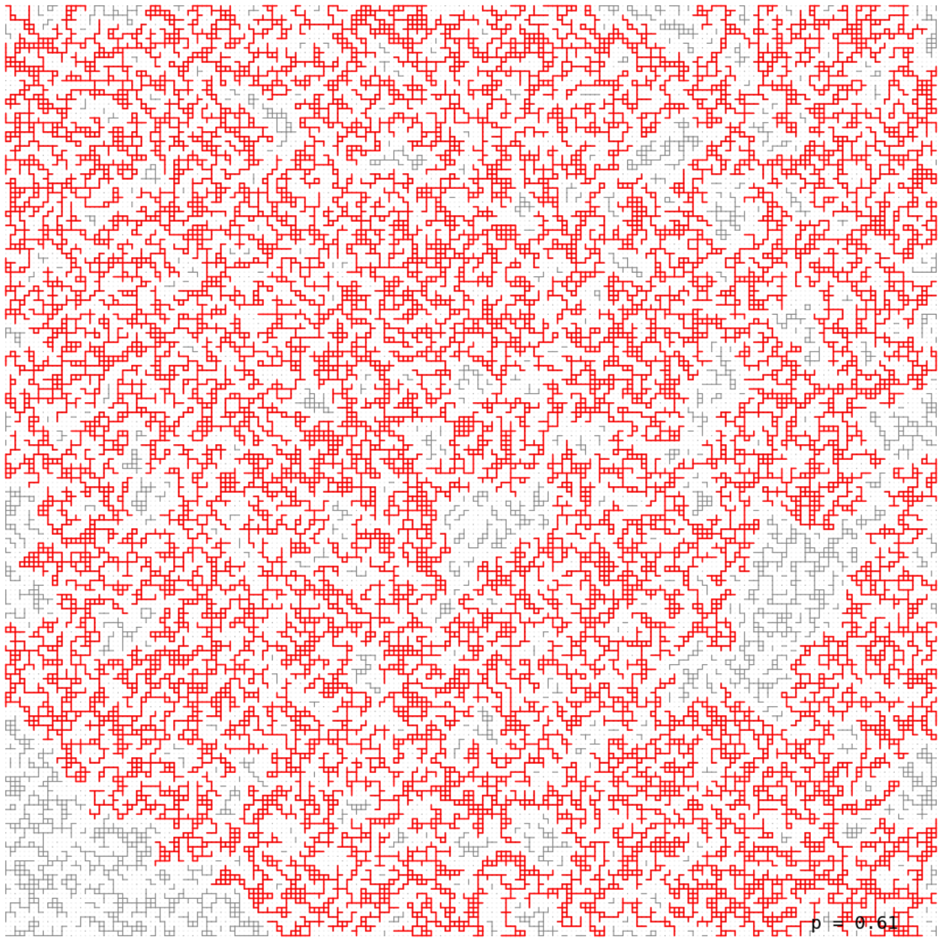

Literature: M.T. Barlow: Random walks and heat kernel on graphs.

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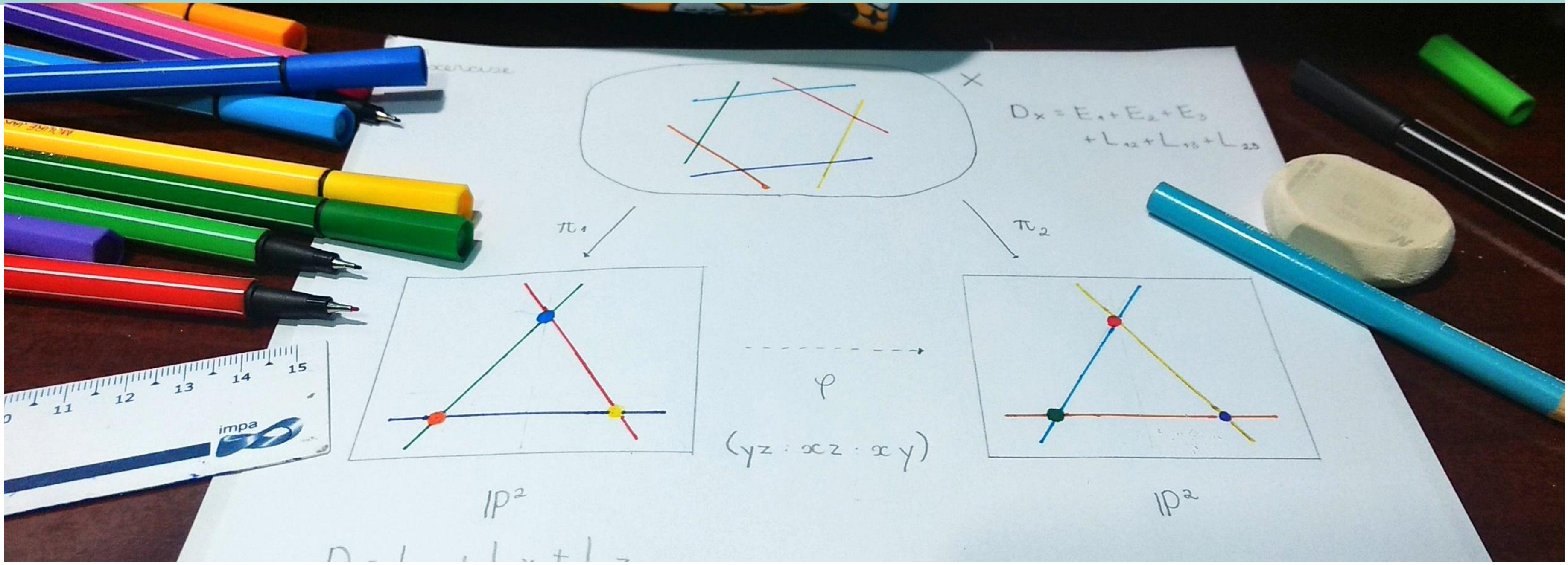
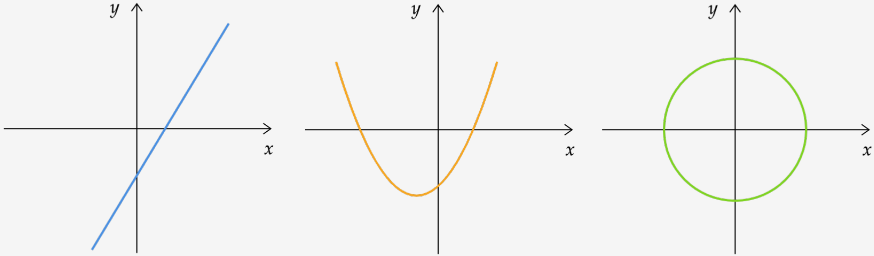
Projective Algebraic Curves and Surfaces I

Dr. Eduardo Alves da Silva



Algebraic Varieties



$$y = ax + b \qquad y = ax^2 + bx + c \qquad x^2 + y^2 = r^2$$

Figure: Remembering old times in school

What is Algebraic Geometry?

- **Affine spaces:**

  $\mathbf{k}$ field (e.g. $\mathbb{R}$ or $\mathbb{C}$) and $n \in \mathbb{N}$

  $\mathbb{A}^n(\mathbf{k}) := \{(x_1, \ldots, x_n) \mid x_i \in \mathbf{k}\}$

- **Affine algebraic varieties:**

  $f_1, \ldots, f_k \in \mathbf{k}[x_0, \ldots, x_n]$ polynomials

  $X = V(f_1, \ldots, f_k) = \{(a_1, \ldots, a_n) \in \mathbb{A}^n(\mathbf{k}) \mid f_i(a_1, \ldots, a_n) = 0 \text{ for all } i\}$

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What is Algebraic Geometry?

- **Rational maps:**

$$\mathbb{A}^n \dashrightarrow \mathbb{A}^m$$
$$(a_1, \dots, a_n) \longmapsto \left(\frac{f_1(x_1, \dots, x_n)}{g_1(x_1, \dots, x_n)}, \dots, \frac{f_m(x_1, \dots, x_n)}{g_m(x_1, \dots, x_n)} \right)$$

$f_1, \dots, f_m, g_1, \dots, g_m \in \mathbf{k}[x_1, \dots, x_n]$ polynomials and $g_i \neq 0$
The map is well-defined outside $V(g_1 \cdots g_m)$.

- $f \colon X \dashrightarrow Y$ rational map between algebraic varieties is a *morphism* if f is defined everywhere.

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What is Algebraic Geometry?

- **Birational equivalence:**

  Given X and Y algebraic varieties, we say that they are *birationally equivalent* if there exists $X \overset{\simeq}{\dashrightarrow} Y$.

  This is equivalent to saying there exist open dense subsets $U \subset X$ and $V \subset Y$ and an isomorphism

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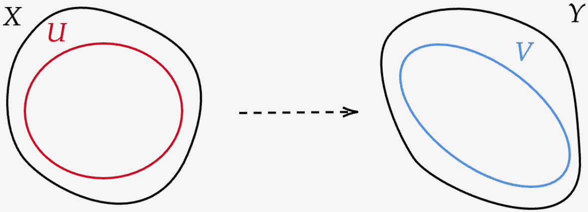
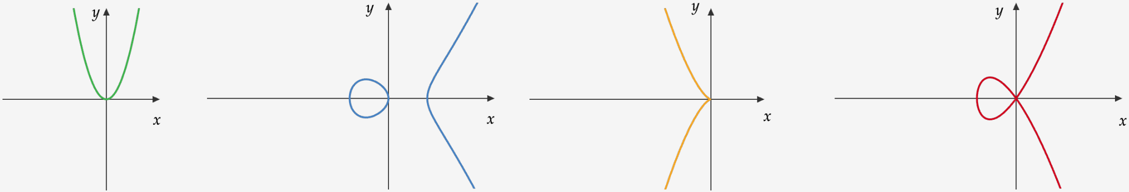


$$X \supset U \xrightarrow{\simeq} V \subset Y$$

Figure: Birational algebraic varieties

Isomorphic x Birational



$A = y - x^2$ $\quad$ $B = y^2 - x^3 + x$ $\quad$ $C = -y^2 - x^3$ $\quad$ $D = y^2 - x^3 - x^2$

Figure: Algebraic curves

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- Affine varieties (more concretely $\mathbf{k}$-sets of affine varieties)
- Projective varieties (again, more concretely $\mathbf{k}$-sets of projective varieties)
- Rational maps and morphisms between varieties
- Dimension and tangential space
- Blow-ups
- Bézout Theorem
- Curves of small degree
- Divisors
- Blow-up of points in $\mathbb{P}^2$

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DANKESCHÖN!
THANK YOU!
OBRIGADO!

DMI